A number of words making complete grammatical structure having sense and meaning is called a sentence. There are two types of sentence:

<table>
<thead>
<tr>
<th>DECLARATIVE SENTENCE</th>
<th>NON-DECLARATIVE SENTENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Declarative sentence makes a statement that declares something (it) gives reliable information/idea.</td>
<td></td>
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<tr>
<td>Example:</td>
<td></td>
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<tr>
<td>(i) Chennai is capital of T.N</td>
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<tr>
<td>(ii) 1+6 = 7</td>
<td></td>
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<tr>
<td>(iii) New Delhi is in England</td>
<td></td>
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<tr>
<td>• Statement that does not declare something (or) give ideas are called non-declarative sentence.</td>
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<tr>
<td>Example:</td>
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<tr>
<td>• Imperative Sentence: Command</td>
<td>Request/Wishes</td>
</tr>
<tr>
<td>• Exclamatory Sentence: !</td>
<td></td>
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<tr>
<td>• Interrogative Sentence: ?</td>
<td></td>
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</tbody>
</table>

PROPOSITION (COR) STATEMENT:
A proposition is a declarative statement which is either TRUE or FALSE but not both.

NON-PROPOSITION:
Sentences having command, questions, exclamations and neither TRUE/FALSE statement.

TRUTH VALUE:
The truth or falsity of a proposition is called truth value.

NOTATIONS:
• If a proposition is true then its truth value is T
• If a proposition is false then its truth value is F
• p, q, r, s, t, etc. are used to denote proposition
### Examples of Propositions with Truth-Value

<table>
<thead>
<tr>
<th>Example</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Five is a prime no.</td>
<td>T</td>
</tr>
<tr>
<td>Dog is a animal</td>
<td>T</td>
</tr>
<tr>
<td>8 ÷ 4 = 2</td>
<td>T</td>
</tr>
<tr>
<td>The earth is round</td>
<td>T</td>
</tr>
<tr>
<td>1 + 2 = 5</td>
<td>F</td>
</tr>
<tr>
<td>Chennai is in England</td>
<td>F</td>
</tr>
<tr>
<td>Delhi is capital of India</td>
<td>T</td>
</tr>
</tbody>
</table>

### Examples of Non-Proposition

<table>
<thead>
<tr>
<th>Example</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>How old are you (?)</td>
<td></td>
</tr>
<tr>
<td>What is height of Himalaya (?)</td>
<td></td>
</tr>
<tr>
<td>The peacock is very beautiful</td>
<td></td>
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<tr>
<td>What a wonderful joke is this (?)</td>
<td></td>
</tr>
<tr>
<td>Obey my order. (Command)</td>
<td></td>
</tr>
<tr>
<td>Please open the door. (Request)</td>
<td></td>
</tr>
<tr>
<td>x + y = 7 [Neither 'T' or 'F']</td>
<td></td>
</tr>
</tbody>
</table>

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### Propositional Logics

The area of logic that deals with proposition is called Proposition Logics.

#### Primary Statements / Atomic Statements

A declarative sentence which cannot be further split up into simple sentences.

Eg: P: Lalitha is a teacher.

#### Connectives / Logical Connectives / Logical Operators

Connectives is an operation which is used to connect two or more than two statements.

Eg: And, or, not.

#### Compound / Molecular / Composite Statements

Statement which contain one or more primary statements and some connectives.

Eg: P: Lalitha is a teacher.
Q: Lalitha teaches Discrete Mathematics.

Compound Statement: Lalitha is a teacher **and** teaches Discrete mathematics.

#### Truth Table

A truth table displays the relationship between the truth values of propositions.

#### Statement Formula

A statement formula is an expression which is a string consisting of variables, parentheses & connective symbols.
**Five Basic Logical Connectives**

### Truth Tables

<table>
<thead>
<tr>
<th>Operator</th>
<th>Truth Table</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>If ... then</strong> (Conditional)</td>
<td></td>
<td>1. If I buy a ticket, I'll go to the movies.</td>
</tr>
<tr>
<td><strong>Or</strong></td>
<td></td>
<td>2. If I don't buy the ticket, I'll go to the movies.</td>
</tr>
<tr>
<td><strong>And</strong></td>
<td></td>
<td>3. If I buy the ticket, I'll go to the movies.</td>
</tr>
<tr>
<td><strong>Not</strong></td>
<td></td>
<td>4. If I don't buy the ticket, I won't go to the movies.</td>
</tr>
</tbody>
</table>

### Examples

1. If I buy a ticket, I'll go to the movies. (P → Q)
2. If I don't buy the ticket, I'll go to the movies. (¬P → Q)
3. If I buy the ticket, I'll go to the movies. (P ∧ Q)
4. If I don't buy the ticket, I won't go to the movies. (¬P ∧ ¬Q)

---

**Additional Information**

- **p**: A proposition.
- **q**: Another proposition.
- **P**: A declarative sentence.
- **Q**: Another declarative sentence.
- **T**: True.
- **F**: False.
- **¬**: Negation.
- **∧**: Conjunction (AND).
- **∨**: Disjunction (OR).
- **→**: Conditional (IF ... then).
- **↔**: Biconditional (IF AND ONLY IF).

---

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**REMARK:** The proposition \( P \rightarrow Q \) & \( P \leftrightarrow Q \) are usually expressed as follows

<table>
<thead>
<tr>
<th>( P \rightarrow Q )</th>
<th>( P \leftrightarrow Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>If P, then Q</td>
<td>( P ) if and only if ( Q )</td>
</tr>
<tr>
<td>P implies Q</td>
<td>( P ) iff ( Q )</td>
</tr>
<tr>
<td>P only if Q</td>
<td>( P ) is necessary and</td>
</tr>
<tr>
<td>( \Theta ) is necessary for ( P )</td>
<td>sufficient for ( \Theta )</td>
</tr>
<tr>
<td>( \Theta ) is sufficient for ( P )</td>
<td>( \Theta )</td>
</tr>
<tr>
<td>Whenever ( P )</td>
<td>Conversely.</td>
</tr>
<tr>
<td>Provided that ( P )</td>
<td>( P \rightarrow \Theta )</td>
</tr>
<tr>
<td>( \Theta ) if ( P )</td>
<td>( \Theta \rightarrow P )</td>
</tr>
<tr>
<td>( \Theta ) is implied by ( P )</td>
<td>( P \rightarrow \Theta )</td>
</tr>
</tbody>
</table>

**SYMBOLIZE THE STATEMENTS USING LOGICAL CONNECTIVES**

1. If either Ram takes C++ or Kumar takes Pascal then Latha will take Lotus.

   **Propositions:**
   - \( P \): Ram takes C++
   - \( Q \): Kumar takes Pascal.
   - \( R \): Latha will take Lotus.

   **Logical Connectives:** \( (P \lor Q) \rightarrow R \)

2. Annu can access the internet from campus only if she is a Computer Science major or she is not a Freshgirl.

   **Propositions:**
   - \( P \): Annu can access the internet from campus.
   - \( Q \): She is a Computer Science major.
   - \( R \): She is a Freshgirl.

   **Logical Connectives:** \( P \rightarrow (Q \lor \neg R) \)

3. (i) If the moon is out & it is not Snowing, then Ram goes out for a walk.
   (ii) If the moon is out, then if it is not Snowing, Ram goes out for a walk.
   (iii) It is not the case that ram goes out for a walk iff it is not Snowing or the moon is out.

   **Propositions:**
   - \( P \): The moon is out
   - \( Q \): It is Snowing
   - \( R \): Ram goes out for a walk

   **Symbolic Expression:**
   - (i) \( (P \land \neg Q) \rightarrow R \)
   - (ii) \( P \rightarrow (\neg Q \rightarrow R) \)
   - (iii) \( \neg (\neg R \leftrightarrow (\neg Q \lor P)) \)

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If P: Menu is rich, q: Menu is happy. Write in symbolic form.

(a) Menu is poor but happy: \( \neg P \land q \)
(b) Menu is rich or unhappy: \( P \lor \neg q \)
(c) Menu is neither rich nor happy: \( \neg P \land \neg q \)
(d) It is necessary for menu to be poor in order to be happy: \( \neg P \rightarrow q \)
(e) Menu to be poor is to be unhappy: \( P \rightarrow \neg q \)
(f) Menu is rich or he is both poor and unhappy: \( P \lor (\neg P \land \neg q) \)

Symbolize the following statements:

(a) If it is raining, then there are clouds in the sky.
(b) If it is not raining, then the sun is not shining and there are clouds in the sky.
(c) The sun is shining if and only if it is not raining.

So: Propositions: \( p \): It is raining
\( q \): There are clouds in the sky
\( r \): The sun is shining

Symbolic expression:

(a) \( p \rightarrow q \)
(b) \( \neg p \rightarrow (\neg q \land q) \)
(c) \( q \rightarrow \neg p \)

H.W.

(a) If it is shining, I shall play tennis in the afternoon.
(b) Finishing the writing of my computer programme before lunch is necessary for playing tennis in this afternoon.
(c) How boundary and sunshine are sufficient to play Tennis in this afternoon.

Express logical connectives as an English sentence:

Let \( p \) and \( q \) be the propositions: "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore" respectively. Express each of these compound propositions as an English sentence:

So: Propositions: \( p \): Swimming at the New Jersey shore is allowed
\( q \): Sharks have been spotted near the shore.

(a) \( \neg q \)  Sharks have not been spotted near the shore.
(b) \( p \land q \)  Swimming at the New Jersey shore is allowed and Sharks have been spotted near the shore.
(c) \( \neg p \lor q \)  Swimming at the New Jersey shore is not allowed or Sharks have been spotted near the shore.
(i) \( p \rightarrow q \): If swimming at the New Jersey shore is allowed then sharks have not been spotted near the shore.

(ii) \( q \rightarrow p \): If sharks have not been spotted near the shore then swimming at the New Jersey shore is allowed.

(iii) \( q \rightarrow p \): If swimming at the New Jersey shore is not allowed then sharks have not been spotted near the shore.

(iv) \( p \leftrightarrow q \): Swimming at the New Jersey shore is allowed iff sharks have not been spotted near the shore.

(v) \( 1p \lor (\neg q) \): Swimming at the New Jersey shore is not allowed and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

Using the following propositions:

\( P \): I finish writing my computer program before lunch.

\( q \): I shall play tennis in afternoon.

\( r \): The sun is shining.

\( s \): The boundary is low.

Express the logical connectives as an English sentence.

(i) \( q \rightarrow p \)

(ii) \( q \leftrightarrow p \)

(iii) \( (s \lor q) \rightarrow p \)

Converse, Contrapositive and Inverse Statements for \( P \) & \( q \) Marks

If \( p \rightarrow q \) is a Conditional statement then

| (i) Converse of \( p \rightarrow q \) | \( q \rightarrow p \) |
| (ii) Contrapositive of \( p \rightarrow q \) | \( \neg q \rightarrow \neg p \) |
| (iii) Inverse of \( p \rightarrow q \) | \( \neg p \rightarrow \neg q \) |

(i) What are the Contrapositive, the Converse and the Inverse of the implication "The home team wins whenever it is raining"?

So: Statement: The home team wins whenever it is raining

Modified Statement: If it is raining then home team wins

(i) Contrapositive of \( p \rightarrow q \): \( \neg q \rightarrow \neg p \)

1a) If the home team does not win then it is not raining

(ii) Converse of \( p \rightarrow q \): \( q \rightarrow p \)

1b) If the home team wins then it is raining

(iii) Inverse of \( p \rightarrow q \): \( \neg p \rightarrow \neg q \)

1c) If it is not raining then the home team does not win
A positive integer is a prime only if it has no divisors other than 1 and itself.

**Solution (Sol):** Proposition: P: A positive integer is a prime. Q: It has no divisors other than 1 and itself.

(i) **Contrapositive of P → Q:**
   - If it has divisors other than 1 and itself then a positive integer is not a prime.

(ii) **Converse of P → Q:**
   - If it has no divisors other than 1 and itself then a positive integer is a prime.

(iii) **Inverse of P → Q:**
   - If a positive integer is not a prime then it has divisors other than 1 and itself.

### Construction of Truth Table

1. How many rows are needed in the truth table of given statement?

   **Solution (Sol):**
   - No. of rows needed in the truth table: \(2^n\) rows, \(n=10\) variable

   - (a) \(P \rightarrow Q: 1\) variable : \(2^1 = 2\) rows
   - (b) \((P \vee Q) \land (Q \vee L)\): 4 variable : \(2^4 = 16\) rows
   - (c) \(Q \lor (P \land Q)\): 6 variable : \(2^6 = 64\) rows

2. Construct the truth table for \((P \lor Q) \lor (P \land Q)\)

   **Solution (Sol):**
   - Since the given statement formula consists of 3 variables, the truth table has \(2^3 = 8\) rows.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>(P \lor Q)</th>
<th>(P \land Q)</th>
<th>((P \lor Q) \lor (P \land Q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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</table>
Construct the truth table for the compound propositions:

\[ T(P \lor Q) \lor \neg T(P \land Q) \lor \neg (P \lor Q) \lor (\neg P \land \neg Q) \]

Sol: No. of variables: 2; No. of rows in truth table: \(2^2 = 4\) rows

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Tp</th>
<th>Tq</th>
<th>(PvQ)</th>
<th>T(PvQ)</th>
<th>T(\neg P \land Q)</th>
<th>T(PvQ) \lor T(\neg P \land Q)</th>
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</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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Construct the truth table for \((P \Rightarrow Q) \iff (R \iff S)\).

Sol: No. of variables: 4; No. of rows in truth table: \(2^4 = 16\) rows

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th>P \Rightarrow Q</th>
<th>R \iff S</th>
<th>(P \Rightarrow Q) \iff (R \iff S)</th>
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<tbody>
<tr>
<td>T</td>
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</table>

Hints:

1. Determine the conditional statement is 'T' or 'F'.
   - (a) \(2 \leq 3\) and \(3\) is a two-digit integer. \(T\)
   - (b) \(\geq 3\) or \(3\) is a two-digit integer. \(T\)
   - (c) If \(a = 3\), then \(10\) is a prime number. \(F\)
   - (d) If \(a = 3\), then \(10\) is a prime number. \(F\)

2. Construct the truth table for:
   - (a) \(T(P \iff Q) \iff (R \iff S)\)
   - (b) \((P \Rightarrow Q) \iff (R \iff S)\)
   - (c) \((P \land Q) \iff (R \iff S)\)
   - (d) \((P \land Q) \iff (R \iff S)\)
# Propositional Equivalence (\(\equiv\))

## Logical Equivalence / Equivalence Rules / Laws of Proposition

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Idempotent Laws</strong></td>
<td>(P \lor P \equiv P)</td>
<td>(P \land P \equiv P)</td>
<td></td>
</tr>
<tr>
<td>2. <strong>Associative Laws</strong></td>
<td>((P \land Q) \land R \equiv P \land (Q \land R))</td>
<td>((P \lor Q) \lor R \equiv P \lor (Q \lor R))</td>
<td></td>
</tr>
<tr>
<td>3. <strong>Commutative Laws</strong></td>
<td>(P \land Q \equiv Q \land P)</td>
<td>(P \lor Q \equiv Q \lor P)</td>
<td></td>
</tr>
<tr>
<td>4. <strong>De Morgan's Laws</strong></td>
<td>(\neg(P \lor Q) \equiv \neg P \land \neg Q)</td>
<td>(\neg(P \land Q) \equiv \neg P \lor \neg Q)</td>
<td></td>
</tr>
<tr>
<td>5. <strong>Distributive Laws</strong></td>
<td>(P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R))</td>
<td>(P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R))</td>
<td></td>
</tr>
<tr>
<td>6. <strong>Complement Laws</strong></td>
<td>(P \lor \neg P \equiv T)</td>
<td>(P \land \neg P \equiv F)</td>
<td></td>
</tr>
<tr>
<td>7. <strong>Dominance Laws</strong></td>
<td>(P \land P \equiv P)</td>
<td>(P \lor P \equiv P)</td>
<td></td>
</tr>
<tr>
<td>8. <strong>Identity Laws</strong></td>
<td>(P \lor F \equiv P)</td>
<td>(P \land T \equiv P)</td>
<td></td>
</tr>
<tr>
<td>9. <strong>Absorption Laws</strong></td>
<td>(P \lor (P \land Q) \equiv P)</td>
<td>(P \land (P \lor Q) \equiv P)</td>
<td></td>
</tr>
<tr>
<td>10. <strong>Double Negation Laws</strong></td>
<td>(\neg\neg P \equiv P)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. <strong>Condition as Disjunction</strong></td>
<td>(P \rightarrow Q \equiv \neg P \lor Q)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. <strong>Biconditional as Condition</strong></td>
<td>(P \iff Q \equiv (P \rightarrow Q) \land (Q \rightarrow P))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. <strong>Exportations Laws</strong></td>
<td>(P \rightarrow (Q \lor R) \equiv (P \rightarrow Q) \lor (P \rightarrow R))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. <strong>Contra Positive Law</strong></td>
<td>(P \rightarrow Q \equiv \neg Q \rightarrow \neg P)</td>
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</tbody>
</table>

## Tautology, Contradiction, and Contingency

### Tautology
- Statement formulae or resultant column is always 'TRUE'

<p>| | | |</p>
<table>
<thead>
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</tbody>
</table>

### Contradiction
- Statement formulae or resultant column is always 'FALSE'

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</tbody>
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### Contingency
- Statement formulae or resultant column is always 'TRUE & FALSE'

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</tbody>
</table>
**Problems to verify tautology, contradiction, and contingency.**

**Method 1: Construction using truth table.**

1. **Prove that \((p \land (p \rightarrow q)) \rightarrow q\) is a tautology.**

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \rightarrow q)</th>
<th>(p \land (p \rightarrow q))</th>
<th>([p \land (p \rightarrow q)] \rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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</tr>
</tbody>
</table>

The resultant column entries are all 'true', the given statement formulae is a tautology. \((p \land (p \rightarrow q)) \rightarrow q\)

2. **Prove that \((\neg p \lor p) \land q\) is a contradiction.**

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(7p)</th>
<th>(7p \lor p)</th>
<th>((\neg p \lor p) \land q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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</tr>
</tbody>
</table>

The resultant column entries are all 'false', the given statement formulae is a contradiction.

3. **Prove that \((p \lor q) \rightarrow (p \lor q)\) is a contingency.**

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \lor q)</th>
<th>(p \rightarrow q)</th>
<th>((p \lor q) \rightarrow (p \lor q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<td>T</td>
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<tr>
<td>T</td>
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<td>F</td>
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<td>F</td>
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<td>T</td>
</tr>
</tbody>
</table>

The resultant column entries show both 'true' \(q\) 'false', the given statement formulae is contingency.

**HW**

1. Show that \((p \land (p \lor q)) \rightarrow q\) is a tautology.
2. Show that \((p \lor q) \rightarrow (p \lor q)\) is a tautology.
3. Verify \((p \lor q) \land q\) \rightarrow p is a tautology.
4. Determine whether \((\neg p \land (p \rightarrow q)) \rightarrow q\) is a tautology.
5. Determine whether \((\neg q \land (p \rightarrow q)) \rightarrow q\) is a tautology.
6. Check whether \((p \land (p \rightarrow q)) \rightarrow (p \lor q)\) is a tautology.
METHOD 2: TO VERIFY TAUTOLOGY USING EQUIVALENCE RULE

1. Show without constructing a truth table for the following.
   (a) \[ (p \land q) \lor (p \land \neg q) \lor (q \land \neg p) \] is a tautology.
   
   Solution:
   \[ \neg \neg (p \land q) \lor (p \land \neg q) \lor (q \land \neg p) \]
   \[ \equiv (p \land q) \lor (p \land \neg q) \lor (q \land \neg p) \]
   \[ \equiv (p \land q) \lor (p \land \neg q) \lor (q \land \neg p) \]
   \[ \equiv (p \land (q \lor \neg q)) \lor (q \land (p \lor \neg p)) \]
   \[ \equiv \top \lor \top \]
   \[ \equiv \top \]

   The given statement formula is a tautology.

2. \[ (q \lor q) \land (p \lor q) \land (p \lor q) \] is a tautology.
   
   Solution:
   \[ \neg \neg (q \lor q) \land (p \lor q) \land (p \lor q) \]
   \[ \equiv (q \lor q) \land (p \lor q) \land (p \lor q) \]
   \[ \equiv (q \lor q) \land (q \lor q) \land (q \lor q) \]
   \[ \equiv (q \lor q) \land (q \lor q) \land (q \lor q) \]
   \[ \equiv \top \land \top \land \top \]
   \[ \equiv \top \]

   The given statement formula is a tautology.

HW:
(a) \( (p \land q) \lor (p \lor q) \lor (q \land \neg p) \) is a tautology.
(b) \( (p \land (q \lor r)) \lor (q \land (p \lor r)) \) is a tautology.
(c) \( (p \land q) \lor (q \land p) \) is a tautology.
(d) \( p \land (r \lor s) \lor (s \land r) \) is a tautology.

DUALITY PRINCIPLE

In dual, if the compound proposition contains \( \land \) and \( \lor \), replace by \( \lor \) and \( \land \) respectively.

Example:
(a) \( S : (p \land q) \land (r \ lor s) \) becomes \( \neg \neg (p \lor q) \lor (r \land s) \)
(b) \( S : (p \land q) \lor (r \land s) \) becomes \( \neg \neg (p \land q) \land (r \lor s) \)
(c) \( S : (p \land q) \lor (r \land s) \) becomes \( \neg \neg (p \lor q) \land (r \land s) \)
(d) \( S : (p \land q) \land (r \lor s) \) becomes \( \neg \neg (p \lor q) \lor (r \land s) \)
**Logical Equivalence:** \((\equiv \text{ or } \equiv)\)

1. **P \equiv Q** is logical equivalence if and only if:
   (i) \(P \equiv Q\) have the same truth value.
   (ii) \(P \equiv Q\) is a tautology.
   (iii) Assume \(P \equiv Q\) (or) Assume \(Q \equiv P\) Derive \(P\).
   (iv) If \(L.H.S = A \equiv R.H.S = B\) then \(A = B \Rightarrow\) logical equivalence.

**Problems on Logical Equivalence**

1. **Show that** \(P \rightarrow q \equiv \top P \lor q\) (or) \(P \rightarrow q \equiv \top P \lor q\) by constructing a truth table.

   **Solution:** Method 1: Constructing using truth table

<table>
<thead>
<tr>
<th>(P)</th>
<th>(q)</th>
<th>(P \rightarrow q)</th>
<th>(\top P)</th>
<th>(\top P \lor q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)</td>
<td>(T)</td>
<td>(F)</td>
<td>(T)</td>
<td>(T)</td>
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<td>(T)</td>
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<td>(F)</td>
<td>(F)</td>
<td>(T)</td>
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</tr>
</tbody>
</table>

   Here, truth values of \(P \rightarrow q\) and \(\top P \lor q\) are the same.

   \(\therefore P \rightarrow q \equiv \top P \lor q\).

2. **Show that** \(P \equiv q \equiv (P \equiv q) \land (q \equiv P) \equiv (\top P \land q) \lor (\top P \land q)\) using truth table.

   **Solution:**

<table>
<thead>
<tr>
<th>(P)</th>
<th>(q)</th>
<th>(P \equiv q)</th>
<th>(\top P)</th>
<th>(\top P \land q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)</td>
<td>(T)</td>
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</table>

   The truth values of 1, 2, 3 are the same.

   \(\therefore P \equiv q \equiv (P \equiv q) \land (q \equiv P) \equiv (\top P \land q) \lor (\top P \land q)\).

3. **Show that** \(P \rightarrow A \equiv \top A \rightarrow \top P\).

   **Solution:** Method 2: \(T P P \rightarrow A \equiv \top A \rightarrow \top P\) is a tautology

   \(\equiv \left[ (P \rightarrow A) \rightarrow (\top A \rightarrow \top P) \right] \land \left[ (\top A \rightarrow \top P) \rightarrow (P \rightarrow A) \right] \)

   \(\equiv \left[ (P \rightarrow A) \rightarrow (\top A \rightarrow \top P) \right] \land \left[ (\top A \rightarrow \top P) \rightarrow (\top P \rightarrow \top A) \right] \)

   \(\equiv \left[ (P \rightarrow A) \rightarrow (\top A \rightarrow \top P) \right] \land \left[ (\top P \rightarrow \top A) \rightarrow (\top P \rightarrow \top A) \right] \)

   \(\equiv \left[ (P \rightarrow A) \right] \land \left[ (\top P \rightarrow \top A) \right] \land \left[ (\top P \rightarrow \top A) \right] \land \left[ (\top P \rightarrow \top A) \right] \)

   \(P \rightarrow q \equiv (P \rightarrow q) \lor (q \rightarrow P) \equiv (\top P \land q) \lor (\top P \land q)\)

   **Simplification**

   \(P \rightarrow q \equiv \top P \lor q\)

   **Simplification**

   \(P \rightarrow q \equiv \top P \lor q\)

   **Distributive Law**
\[ \equiv [(\neg P) \land (\neg Q)] \land [(\neg Q) \land (\neg P)] \]
\[ \equiv (\neg P) \land (\neg Q) \]
\[ \equiv \top \land \top \]
\[ \equiv \top \]

H.W. ④ Show that \( P \rightarrow (Q \rightarrow P) \iff \top \rightarrow (P \rightarrow Q) \iff [(P \land Q) \rightarrow \top] \)

Sol: Method ⑥ Assume L.H.S. \( Q \) and derive R.H.S.

① L.H.S. = \( P \rightarrow (Q \lor Q) \equiv \top \lor (Q \lor Q) \)
\[ \equiv (Q \lor Q) \lor \top \]
\[ \equiv (Q \lor Q) \lor \top \]
\[ \equiv Q \lor Q \]
\[ \equiv \top \]
\[ \equiv R.H.S. \]

H.W. ⑤ Show that \( (P \land (Q \land Q)) \lor (Q \land (P \land Q)) \iff \top \cdot \]

Sol: L.H.S. = \( (P \land (Q \land Q)) \lor (Q \land (P \land Q)) \)
\[ \equiv [(P \land Q) \land Q] \lor [(Q \land P) \land Q] \]
\[ \equiv [(P \land Q) \land Q] \lor [(Q \land P) \land Q] \]
\[ \equiv [(P \land Q) \land Q] \lor [(Q \land P) \land Q] \]
\[ \equiv \top \land Q \]
\[ \equiv R.H.S. \]

H.W. Show that

① \( (P \land Q) \rightarrow (P \lor (Q \land Q)) \iff (P \lor Q) \)
② \( (P \land Q) \rightarrow (P \lor (Q \land Q)) \iff (P \lor Q) \)
③ \( (Q \land (P \lor Q)) \land Q \iff (P \lor Q) \land Q \)
④ \( (Q \land (P \lor Q)) \land Q \iff (P \lor Q) \land Q \)
⑤ \( (Q \land (P \lor Q)) \land Q \iff (P \lor Q) \land Q \)
⑥ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑦ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑧ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑨ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑩ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑪ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑫ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑬ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑭ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑮ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑯ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑰ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑱ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑲ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑳ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑳ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
⑳ \( (P \land Q) \land (Q \land P) \land (P \land Q) \land (Q \land P) \)
Show that $P \rightarrow (q \rightarrow P) \iff \neg P \rightarrow (P \rightarrow q)$

**Solution:** Method 1: If L.H.S = A & R.H.S = B then A = B $\iff$ Logical Equivalent

<table>
<thead>
<tr>
<th>L.H.S: $P \rightarrow (q \rightarrow P)$</th>
<th>R.H.S: $\neg P \rightarrow (P \rightarrow q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\equiv T \lor (T \lor P)$</td>
<td>$\equiv \neg P \lor (P \lor q)$</td>
</tr>
<tr>
<td>$\equiv T \lor q \lor P$</td>
<td>$\equiv P \land q$</td>
</tr>
<tr>
<td>$\equiv T \lor q$</td>
<td></td>
</tr>
<tr>
<td>$\equiv T$</td>
<td></td>
</tr>
</tbody>
</table>

Here L.H.S = R.H.S, \( \therefore \) The given statements are equivalent.

Prove the following equivalences by proving the equivalence of the dual $T[(\neg P \land q) \land (\neg P \land T \land q)] \land [P \lor q] \equiv P$.

**Solution:**

**Statement Formula:** $T[(\neg P \land q) \land (\neg P \land T \land q)] \land [P \lor q] \equiv P$

**Dual Statement Formula:** $T[\neg (P \lor q) \lor (\neg P \land q)] \land [P \lor q] \equiv P$

<table>
<thead>
<tr>
<th>L.H.S: $T[\neg (P \lor q) \lor (\neg P \land q)] \land [P \lor q]$</th>
<th>R.H.S: $T[(\neg P \land q) \land (\neg P \land T \land q)] \land [P \lor q]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\iff [(P \land T \land q)] \land (P \lor q)$</td>
<td>$\iff [T \lor P \land (\neg P \lor q)]$</td>
</tr>
<tr>
<td>$\iff (P \land T \land q) \land (P \lor q)$</td>
<td>$\iff (P \land T \land q) \land (P \lor q)$</td>
</tr>
<tr>
<td>$\iff P \land (P \lor q)$</td>
<td>$\iff P \land (P \lor q)$</td>
</tr>
<tr>
<td>$\iff P$</td>
<td>$\iff P$</td>
</tr>
</tbody>
</table>

\( \therefore \) $T[(\neg P \land q) \land (\neg P \land T \land q)] \land [P \lor q] \equiv P$

H.W: Solve the following using Truth Table or Laws of Logic

(a) $\neg (P \lor q) \land (P \land (P \lor q)) \equiv P \lor q$

(b) $T \lor (q \rightarrow P) \equiv q \rightarrow (P \lor q)$

(c) $T \lor (P \rightarrow q) \equiv q \rightarrow (P \lor q)$

(d) $P \rightarrow q \lor (T \rightarrow q) \equiv 0$

(e) $[\neg (P \rightarrow q) \land (P \land (P \lor q))] \equiv P \lor q$

(f) $(P \land q) \lor (P \land (P \lor q)) \equiv (P \land q)$

(g) $(T \lor q \lor P) \rightarrow (P \lor (P \lor q)) \equiv T$

(h) $(T \lor (P \land q)) \lor (P \land (P \lor q)) \equiv T$

(i) $(P \land q) \land (T \lor (P \land q)) \equiv T$

(j) $(T \lor (P \land q)) \lor (P \land (P \lor q)) \equiv T$

(k) $(P \land q) \land (T \lor (P \land q)) \equiv T$

(l) $(T \lor (P \land q)) \lor (P \land (P \lor q)) \equiv T$

(m) $(P \land q) \land (T \lor (P \land q)) \equiv T$

(n) $(T \lor (P \land q)) \lor (P \land (P \lor q)) \equiv T$
PROBLEMS ON TAUTOLOGICAL IMPLICATION (⇒)

TAUTOLOGICAL IMPLICATION: \( P \implies Q \) iff \( R\implies S \) is a Tautology.

1. Prove that \( (p \implies (q \implies r)) \implies ((p \implies q) \implies (p \implies r)) \) by constructing the Truth Table.

   \[ \begin{array}{|c|c|c|c|c|c|c|c|c|c|}
   \hline
   P & q & r & q\implies r & p \implies (q \implies r) & p \implies q & p \implies r & (p \implies q) \implies (p \implies r) & (p \implies (q \implies r)) \implies ((p \implies q) \implies (p \implies r)) \\
   \hline
   T & T & T & T & T & T & T & T & T \\
   T & T & F & F & F & F & F & F & T \\
   T & F & T & T & T & T & T & T & T \\
   T & F & F & F & F & F & F & T & T \\
   F & T & T & T & T & T & T & T & T \\
   F & T & F & F & T & F & T & F & T \\
   F & F & T & T & T & T & T & T & T \\
   F & F & F & F & T & T & T & T & T \\
   \hline
   \end{array} \]

   The statement formulae is a Tautology Implication.

2. Show that the following implications without constructing the T-T:
   \[ [(p \lor q) \implies r] \implies [(p \lor q) \implies r] \implies (q \implies r) \]

   \[ \text{Given, Complementary Law} \]

   \[ [(p \land q) \implies (p \lor q)] \implies (q \implies (p \lor q)) \]

   \[ \text{Complementary Law} \]

   \[ [(p \lor q) \implies (p \land q)] \implies (q \implies (p \land q)) \]

   \[ \text{Complementary Law} \]

   \[ [(p \land q) \implies (q \lor p)] \implies (q \implies (q \lor p)) \]

   \[ \text{Complementary Law} \]

   \[ [(p \lor q) \implies (q \lor p)] \implies (q \implies (q \lor p)) \]

   \[ \text{Complementary Law} \]

   \[ [(p \land q) \implies (p \lor q)] \implies (q \implies (p \lor q)) \]

   \[ \text{Complementary Law} \]

   \[ (p \land q) \implies (p \lor q) \]

   \[ \text{Complementary Law} \]

3. Show that \( (q \implies (p \land q)) \implies (q \implies (p \lor q)) \implies (q \implies q) \).

   \[ \text{Given, Complementary Law} \]

   \[ [(q \implies (p \land q)) \implies (q \implies (p \lor q))] \implies (q \implies q) \]

   \[ \text{Tautology} \]
Complementary Law
Conditional Law
Identity Law
Conditional Law
Double negation law
Commutative law

\[ (\neg q \vee \neg p) \Rightarrow (\neg q \vee \neg p) \]
\[ \Rightarrow (q \Rightarrow q) \]
\[ \Rightarrow (q \Rightarrow q) \]
\[ \Rightarrow (\neg p \vee q) \Rightarrow (\neg p \vee q) \]
\[ \Rightarrow (p \vee (\neg p \vee q)) \Rightarrow (p \vee (\neg p \vee q)) \]
\[ \Rightarrow (p \vee q) \Rightarrow (p \vee q) \]
\[ \Rightarrow (q \Rightarrow q) \]
\[ \Rightarrow (q \Rightarrow q) \]
\[ \Rightarrow (q \Rightarrow q) \]
\[ \Rightarrow (q \Rightarrow q) \]

Since \[ [(q \Rightarrow (p \vee q)) \Rightarrow (p \vee q)] \Rightarrow (q \Rightarrow q) \] is a tautology.

\[ \Rightarrow [(q \Rightarrow (p \vee q)) \Rightarrow (p \vee q)] \]

4. Show that \((p \Rightarrow \alpha) \land (q \Rightarrow \alpha) \Rightarrow (p \lor q) \Rightarrow \alpha\).

Sol.: To prove the tautological implication it is enough to prove

\[ [(p \Rightarrow \alpha) \land (q \Rightarrow \alpha)] \Rightarrow [(p \lor q) \Rightarrow \alpha] \]

\[ \Rightarrow [(1 \lor \alpha) \land (1 \lor \alpha)] \Rightarrow [1 \lor (p \lor q)] \lor \alpha] \]

\[ \Rightarrow [(1 \lor \alpha) \land (1 \lor \alpha)] \Rightarrow [1 \lor (p \lor q)] \lor \alpha] \]

\[ \Rightarrow [1 \lor (p \lor q)] \lor \alpha] \]

\[ \Rightarrow [1 \lor (p \lor q)] \lor \alpha] \]

\[ \Rightarrow 1 \]

Since \[ [(p \Rightarrow \alpha) \land (q \Rightarrow \alpha)] \Rightarrow (p \lor q) \Rightarrow \alpha \]

\[ \Rightarrow [(p \Rightarrow \alpha) \land (q \Rightarrow \alpha)] \Rightarrow (p \lor q) \Rightarrow \alpha \]

H.W.

Show that

(a) \( p \Rightarrow (p \Rightarrow q) \)
(b) \( (p \land q) \Rightarrow p \lor q \)
(c) \( (p \Rightarrow q) \land (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \)
(d) \( (p \Rightarrow q) \land (p \Rightarrow q) \Rightarrow tp \)
(e) \( \neg p \Rightarrow (p \Rightarrow q) \)

\[ p \Rightarrow (p \Rightarrow q) \]

\[ p \Rightarrow q \]
If we write a given statement in terms of $\land$, $\lor$ and $\neg$ then it is called
NORMAL FORM / CANONICAL FORM

<table>
<thead>
<tr>
<th>PRINCIPAL DISJUNCTIVE NORMAL FORM (PDNF)</th>
<th>PRINCIPAL CONJUNCTIVE NORMAL FORM (PCNF)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CONJUNCTION</strong> : $\land$ (PRODUCT)</td>
<td><strong>DISJUNCTION</strong> : $\lor$ (SUM)</td>
</tr>
<tr>
<td>Elementary Product:</td>
<td>Elementary Sum:</td>
</tr>
<tr>
<td>$\land$ Product of variables and their Negation.</td>
<td>The sum of variables and their Negation.</td>
</tr>
<tr>
<td>$\neg P \land P, \neg Q \land Q, P \land \neg P, Q \land \neg Q,$</td>
<td>$P \lor Q, \neg P \lor Q, P \land \neg Q, \neg P \land Q,$</td>
</tr>
<tr>
<td>$\neg P \land \neg Q, P \land Q, P \land Q, Q \land P,$</td>
<td>...</td>
</tr>
</tbody>
</table>

**Disjunctive Normal Form, DNF:**

$\text{DNF} = (\text{Elementary}) \lor (\text{Elementary}) \lor (\text{Elementary})$

$\text{DNF} = (P \lor Q) \lor (P \lor \neg Q) \lor (\neg P \lor Q)$ is an Example.

**Conjunctive Normal Form, CNF:**

$\text{CNF} = (\text{Elementary}) \land (\text{Elementary}) \land (\text{Elementary})$

$\text{CNF} = (P \land Q) \land (P \land \neg Q) \land (\neg P \land Q)$.

**Minterms :** Formulas consisting of conjunction of $P$ and $\neg P$ but not both of the same variables and its Negation.

- Minterms of 2 variables: $2^2 = 4$ terms
  $P \land \neg P, \neg P \land P, P \land P, \neg P \land \neg P$
- Minterms of 3 variables: $2^3 = 8$ terms
  $P \land Q \land \neg R, P \land \neg Q \land R, P \land \neg Q \land \neg R,$
  $\neg P \land Q \land R, \neg P \land Q \land \neg R, \neg P \land \neg Q \land R,$
  $\neg P \land \neg Q \land \neg R.$

**DNF :** SUM of minterms

$\text{DNF} = (P \lor Q) \lor (P \lor \neg Q) \lor (\neg P \lor Q)$

**WORKING RULE TO OBTAIN PDNF**

Step 1: Write the given statement in terms of $\land , \lor$ alone.
Step 2: Apply (Each term) $\land T$. ($\land \neg T \equiv \neg P$)
Step 3: Instead of $\lor$, apply $P \land \neg Q$.
Step 4: Apply Distributive Law
Step 5: Apply Commutative Law

**NOTE:** DNF & CNF are not Unique.

**WORKING RULE TO OBTAIN PCNF**

Step 1: Write the given statement in terms of $\land , \lor$ alone.
Step 2: Apply (Each term) $\lor T$. ($\lor \neg T \equiv P$)
Step 3: Instead of $\land$, apply $P \land \neg Q$.
Step 4: Apply Distributive Law
Step 5: Apply Commutative Law

**NOTE:** PDNF and PCNF are Unique.
Find PDNF and PCNF of the following compound proposition using truth table and laws of proposition.

\[(T \lor q) \implies (p \iff q)\]

**So:** USING TRUTH TABLE

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>Tp</th>
<th>Tq</th>
<th>Tp\lor q</th>
<th>P\lor q</th>
<th>(Tp\lor q)\iff(p\iff q)</th>
<th>MINTERMS</th>
<th>MAXTERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
<td>T</td>
<td>(pq)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td></td>
<td>T</td>
<td>(pq)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td></td>
<td>T</td>
<td>(pq)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td>(Tp\lor q)</td>
<td></td>
<td>PVq</td>
</tr>
</tbody>
</table>

PDNF: \(S = (pq) \lor (pq) \lor (pq)\)

PCNF: \(P \lor q\)

Using laws of propositions,

Let \(S = (T \lor q) \implies (p \iff q)\)

\[
= (T \lor q) \lor (p \iff q)
\]

\[
= (p \lor q) \lor [(p \lor q) \lor (T \lor q)]
\]

PDNF of \(S = (pq) \lor (pq) \lor (pq)\)

PCNF of \(S = T (PDNF \lor QS) \lor (T \lor q) = PVq\)

PDNF: \(S = (pq) \lor (pq) \lor (pq)\)

PCNF: \(P \lor q\)

\(\text{Conditional law:} \quad \text{Demorgan's Law:} \quad (p \iff q) = (pq) \lor (pq)
\]

6. \((T \lor p) \land (q \iff p) \lor p\).

So: USING TRUTH TABLE

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>R</th>
<th>TP</th>
<th>TP\lor R</th>
<th>q\iff p</th>
<th>(T \lor p) \land (q \iff p)</th>
<th>MINTERM</th>
<th>MAXTERM</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>(pq)</td>
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<td>T</td>
<td>T</td>
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<td>T</td>
<td>T</td>
<td>(p \lor q) \lor (p \lor q)</td>
<td></td>
<td>PVq</td>
</tr>
</tbody>
</table>

PDNF: \(S = (pq) \lor (pq) \lor (pq)\)

PCNF: \(P \lor q\)


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USING LAWS OF PROPOSITION

Let \( S = (T \rightarrow R) \land (Q \rightarrow P) \)

\[
\begin{align*}
S &= (T \rightarrow P) \land (Q \rightarrow P) \\
&= (T \land \neg P) \land (Q \land \neg P) \\
&= \neg (T \lor Q) \\
&= \neg T \lor \neg Q
\end{align*}
\]

\( P \lor (Q \lor P) \)

\( P \lor (Q \lor P) = (P \lor Q) \lor P \)

Distributive Law

\[ P \lor (Q \lor P) = (P \lor Q) \lor (P \lor Q) \]

\( P \lor (Q \lor P) = (Q \lor P) \lor (P \lor Q) \)

\( P \lor (Q \lor P) = (Q \lor P \lor Q) \lor (P \lor Q) \)

\( P \lor (Q \lor P) = (Q \lor P) \lor (Q \lor P) \)

\( P \lor (Q \lor P) = (Q \lor P) \lor (Q \lor P) \)

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\( P \lor (Q \lor P) = (Q \lor P) \lor (Q \lor P) \)

\( P \lor (Q \lor P) = (Q \lor P) \lor (Q \lor P) \)
4. Obtain PDNF & hence PCNF of \((\neg P \land Q \land R) \lor (\neg P \land Q) \lor (\neg Q \land \neg R)\)
Sol: Let \(S = (\neg P \land Q) \lor (\neg P \land Q) \lor (\neg Q \land \neg R)\)

\[S = (\neg P \land \neg Q \land \neg R) \lor (\neg P \land Q) \lor (\neg P \land Q) \lor (\neg Q \land \neg R)\]

5. Find PCNF and PDNF of \((P \rightarrow (Q \land R)) \land (T \rightarrow (Q \lor R))\)
Sol: \((P \rightarrow (Q \land R)) \land (T \rightarrow (Q \lor R))\)

\[\neg (P \land \neg (Q \land R)) \land \neg (T \land \neg (Q \lor R))\]

Condition due: Distributive law.
Complementary law
Complementary law
Distributive law.
6. Obtain PDNF & hence PCNF of $P \rightarrow (P \rightarrow Q) \land (Q \lor \neg P)$

Sol: $P \rightarrow (P \rightarrow Q) \land (Q \lor \neg P)$

$\equiv P \rightarrow ((P \rightarrow Q) \land (Q \lor \neg P))$

$\equiv P \rightarrow ((\neg P \lor Q) \land (Q \lor \neg P))$

$\equiv P \rightarrow ((Q \lor \neg P) \land (Q \lor \neg P))$

$\equiv P \rightarrow ((Q \lor \neg P) \lor (Q \lor \neg P))$

$\equiv P \rightarrow (Q \lor \neg P)$

$\equiv T \lor (Q \lor \neg P)$

$\equiv (Q \lor \neg P)$

$\equiv (Q \land (Q \lor \neg P)) \lor (Q \land (Q \lor \neg P))$

PDNF: $S \equiv (Q \land (Q \lor \neg P)) \lor (Q \land (Q \lor \neg P))$

PCNF: $T \lor (Q \lor \neg P)$

PDNF, $T \equiv P \rightarrow Q$

PCNF, $S \equiv T \lor (Q \lor \neg P)$

De Morgan's Law
Condition Law
Commutative Law
Distributive Law
Complementary Law
Identity Law
Identity Law
Condition Law
PDNF $\equiv P$
DVTP $\equiv T$
Distributive Law

4. Obtain PDNF for $P \rightarrow (\neg P \land (Q \Rightarrow P))$

Sol: $P \rightarrow (\neg P \land (Q \Rightarrow P))$

$\equiv T \lor (\neg P \land (Q \Rightarrow P))$

$\equiv (\neg P \land (Q \Rightarrow P))$

$\equiv (\neg P \land (Q \land (Q \lor \neg P)))$

$\equiv (\neg P \land (Q \land (Q \lor \neg P)))$

$\equiv (\neg P \land (Q \land (Q \lor \neg P)))$

PDNF: $S \equiv (\neg P \land (Q \land (Q \lor \neg P)))$

PDNF, $T \equiv P \rightarrow Q$

PDNF, $S \equiv (\neg P \land (Q \land (Q \lor \neg P)))$

PCNF, $S \equiv (\neg P \land (Q \land (Q \lor \neg P)))$

Condition Law
Distributive Law
PDNF $\equiv T$

4. Obtain PCNF of $P \rightarrow (P \land (Q \Rightarrow P))$

Sol: $P \rightarrow (P \land (Q \Rightarrow P))$

$\equiv T \lor (P \land (Q \Rightarrow P))$

$\equiv (P \land (Q \Rightarrow P))$

$\equiv (P \land (Q \lor P))$

$\equiv (P \land (Q \lor P))$

$\equiv (P \land (Q \lor P))$

PDNF, $S \equiv (P \land (Q \lor P))$

PDNF, $T \equiv P \rightarrow Q$

PDNF, $S \equiv (P \land (Q \lor P))$

PCNF, $S \equiv (P \land (Q \lor P))$

Condition Law
Distributive Law
Idempotent Law
Associative Law
Complementary Law

PDNF, $S \equiv (P \land (Q \lor P))$

PDNF, $T \equiv P \rightarrow Q$

PDNF, $S \equiv (P \land (Q \lor P))$

PCNF, $S \equiv (P \land (Q \lor P))$

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Condition Law
Distributive Law
Idempotent Law
Associative Law
Complementary Law

PDNF, $S \equiv (P \land (Q \lor P))$

PDNF, $T \equiv P \rightarrow Q$

PDNF, $S \equiv (P \land (Q \lor P))$

PCNF, $S \equiv (P \land (Q \lor P))$
**HW:** Find DNF and CNF for the following statement formulae
(a) \( P \land (P \rightarrow Q) \)  (b) \((P \rightarrow (Q \lor P)) \land (P \rightarrow (Q \land P))\)
(c) \((P \lor Q) \land (P \rightarrow Q)\)  (d) \((Q \land P) \lor (Q \land P)\)  (e) \(P \land (P \rightarrow Q)\)

**Few More Logical Equivalence**

1. \((P \rightarrow Q) \land (Q \rightarrow R) \equiv (P \rightarrow R) \lor (Q \rightarrow P)\)
2. \((P \rightarrow Q) \lor (Q \rightarrow P) \equiv (P \lor Q) \lor (Q \lor P)\)
3. \((P \rightarrow Q) \land (P \rightarrow R) \equiv P \rightarrow (Q \land R)\)

**3. Rules of Inference**

**Theorem:** Premises/Hypothesis: When all premises are assumed to be True.

**Conclusion:** Then conclusion is also True.

**Proof:**
- Direct Proof
- Indirect Proof
- Conditional Proof

**Rules of Inference | Implication Rule | Rules for Valid Conclusion**

<table>
<thead>
<tr>
<th><strong>Name</strong></th>
<th><strong>Formula</strong></th>
<th><strong>Premise</strong></th>
<th><strong>Conclusion</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. MODUS PONENS</td>
<td>( P \rightarrow Q )</td>
<td>( P, P \rightarrow Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td>2. MODUS TOLLENS</td>
<td>( P \rightarrow Q )</td>
<td>( \neg Q, P \rightarrow Q )</td>
<td>( \neg P )</td>
</tr>
<tr>
<td>3. ADDITION</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
</tr>
<tr>
<td>4. CONJUNCTION</td>
<td>( P \rightarrow Q )</td>
<td>( P \rightarrow Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td>5. SIMPLIFICATION</td>
<td>( P \land Q )</td>
<td>( P \land Q )</td>
<td>( P \land Q )</td>
</tr>
<tr>
<td>6. DISJUNCTION</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
</tr>
<tr>
<td>7. HYPOTHETICAL SYLLOGISM</td>
<td>( P \rightarrow Q )</td>
<td>( P \rightarrow Q )</td>
<td>( P \rightarrow Q )</td>
</tr>
<tr>
<td>8. RESOLUTION</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
</tr>
<tr>
<td>9. DILEMMA</td>
<td>( P \lor Q )</td>
<td>( P \lor Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td>10. TACIT</td>
<td>( P \rightarrow Q )</td>
<td>( P \rightarrow Q )</td>
<td>( Q )</td>
</tr>
</tbody>
</table>
**DIRECT PROOF**

When a conclusion is derived from a set of premises by using the equivalence rule and implication rule, then the process of derivation is called a direct proof.

1. **Show that TP is a valid conclusion from the premises TPvQ, T(QvR) & TR using logical implication.**

   **Solution:**
   - **Premises:** TPvQ, T(QvR), TR
   - **Conclusion:** TP [Direct Proof]

   **Steps** | **Premise** | **Reason**
   --- | --- | ---
   1 | T(QvR) | Rule P
   2 | TQvTR | (1, Rule T: Demorgan's Law)
   3 | TQ | (2, Rule T: Simplification Law)
   4 | TPvQ | Rule P
   5 | TP | §3.4.3, Rule T: Disjunctive Syllogism

   **Conclusion:** TP is a valid conclusion.

2. **Show that T is a valid conclusion from the premises P→Q, Q→R, R→S, TS, and PVT.**

   **Solution:**
   - **Premises:** P→Q, Q→R, R→S, TS, PVT
   - **Conclusion:** T

   **Steps** | **Premises** | **Reason**
   --- | --- | ---
   ① | P→Q | Rule P
   ② | Q→R | Rule P
   ③ | P→R | Rule P
   ④ | R→S | Rule P
   ⑤ | P→S | Rule P
   ⑥ | TS | §3.4.3, Rule T: Hypothetical
   ⑦ | TP | §3.4.3, Rule T: Hypothetical
   ⑧ | PVT | §5.6.3, Rule T: Modus Tollens
   ⑨ | T | §7.8.3, Rule T: Disjunction

   **Conclusion:** T is a valid conclusion.

3. **Show that S is a valid conclusion from the premises P→Q, P→R, T(QvR) & SVP.**

   **Solution:**
   - **Premises:** P→Q, P→R, T(QvR), SVP
   - **Conclusion:** S [Direct Proof]

   **Steps** | **Premise** | **Reason**
   --- | --- | ---
   | | | Clue: SVP
   | | | SVP
   | | | TP
   | | | §1.3

   **Conclusion:** S
### Steps and Premises

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \neg (q \land r) )</td>
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</tr>
<tr>
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</tr>
<tr>
<td>4</td>
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<td>Rule P</td>
</tr>
<tr>
<td>5</td>
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<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( p \Rightarrow q )</td>
<td>Rule P</td>
</tr>
<tr>
<td>7</td>
<td>( q \Rightarrow r )</td>
<td>Rule P</td>
</tr>
<tr>
<td>8</td>
<td>( p \Rightarrow q )</td>
<td>Rule P</td>
</tr>
<tr>
<td>9</td>
<td>( p \lor q )</td>
<td>Rule P</td>
</tr>
<tr>
<td>10</td>
<td>( q )</td>
<td>Rule P</td>
</tr>
<tr>
<td>11</td>
<td>( s \lor p )</td>
<td>Rule P</td>
</tr>
<tr>
<td>12</td>
<td>( s )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

### Conclusion
- \( s \) is a valid conclusion

### Reasoning

1. **Premises**
   - \( p \Rightarrow q \)
   - \( q \lor r \)
   - \( q \Rightarrow r \)
   - \( p \Rightarrow q \)

2. **Conclusion**
   - \( s \) or \( p \)

3. **Reasoning**
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P
   - Rule P

4. **Final Conclusion**
- \( s \) is a valid conclusion

---

5. **Show that** \( (p \Rightarrow q) \land (q \lor r) \land (r \Rightarrow s) \) \( \Rightarrow \) \( s \) is a tautologically implied by

6. **Premises**
   - \( p \lor q \lor r \)
   - \( \neg p \lor \neg q \lor \neg s \)

7. **Conclusion**
- \( s \lor r \)

---

---

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### Show that RVS follows tautology from the following Premises.

$$CVD, CVD \rightarrow TH, TH \rightarrow (A \land TB) \land (A \land TB) \rightarrow RVS$$

**Solution:**

Premises: $$CVD, CVD \rightarrow TH, TH \rightarrow (A \land TB), (A \land TB) \rightarrow RVS$$

Conclusion: RVS

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>CVD</td>
<td>Rule P</td>
</tr>
<tr>
<td>②</td>
<td>CVD → TH</td>
<td>Rule P</td>
</tr>
<tr>
<td>③</td>
<td>TH</td>
<td>$\text{§2, 13 ; Rule T: Modus Ponens}$</td>
</tr>
<tr>
<td>④</td>
<td>TH → (A ∧ TB)</td>
<td>Rule P</td>
</tr>
<tr>
<td>⑤</td>
<td>A ∧ TB</td>
<td>$\text{§4, 3 ; Rule T: Modus Ponens}$</td>
</tr>
<tr>
<td>⑥</td>
<td>A ∧ TB → RVS</td>
<td>$\text{§6, 5 ; Rule T: Modus Ponens}$</td>
</tr>
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</table>

... RVS is a valid conclusion.

### Show that $P \rightarrow (Q \land (R \rightarrow S))$, $(Q \land M) \land (S \rightarrow N)$, $T(M \land N)$, $P \rightarrow R \Rightarrow TP$.

**Solution:**

Premises: $P \rightarrow (Q \land (R \rightarrow S))$, $(Q \land M) \land (S \rightarrow N)$, $T(M \land N)$, $P \rightarrow R$.

Conclusion: TP.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>$P \rightarrow (Q \land (R \rightarrow S))$</td>
<td>Rule P</td>
</tr>
<tr>
<td>②</td>
<td>$P \rightarrow Q$</td>
<td>$\text{§17 ; Rule T: Simplification}$</td>
</tr>
<tr>
<td>③</td>
<td>$R \rightarrow S$</td>
<td>$\text{§17 ; Rule T: Simplification}$</td>
</tr>
<tr>
<td>④</td>
<td>$(Q \land M) \land (S \rightarrow N)$</td>
<td>Rule P</td>
</tr>
<tr>
<td>⑤</td>
<td>$Q \land M$</td>
<td>$\text{§4, 3 ; Rule T: Simplification}$</td>
</tr>
<tr>
<td>⑥</td>
<td>$S \rightarrow N$</td>
<td>$\text{§4, 3 ; Rule T: Simplification}$</td>
</tr>
<tr>
<td>⑦</td>
<td>$P \rightarrow R$</td>
<td>Rule P</td>
</tr>
<tr>
<td>⑧</td>
<td>$P \rightarrow S$</td>
<td>$\text{§7, 3 ; Rule T: Hypothetical Syll.}$</td>
</tr>
<tr>
<td>⑨</td>
<td>$P \rightarrow N$</td>
<td>$\text{§8, 6 ; Rule T: Hypothetical Syll.}$</td>
</tr>
</tbody>
</table>
(1) \( y \equiv \{x(y)\} \)

(2) \( \neg M \lor \neg N \)
(3) \( \neg N \lor \neg M \)

\[ \text{Rule P} \]

(4) \( N \Rightarrow \neg M \)

(5) \( P \Rightarrow \neg M \)

(6) \( M \Rightarrow \neg P \)

(7) \( M \)

(8) \( \neg P \)

\[ \text{TP is a valid conclusion.} \]

8. Show that the hypothesis \((p \land q) \lor r \land s\) and \(r \Rightarrow s\) imply PVS

Premises: \((p \land q) \lor r \land s\) \(r \Rightarrow s\)

Conclusion: PVS

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((p \land q) \lor r \land s)</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>((p \land q) \land (q \lor r))</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>(p \land q)</td>
<td>(\neg 123), Rule T: Distributive Law</td>
</tr>
<tr>
<td>4</td>
<td>(q \lor r)</td>
<td>(\neg 23), Rule T: Simplification Law</td>
</tr>
<tr>
<td>5</td>
<td>(r \Rightarrow s)</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>T \lor \neg s</td>
<td>(\neg 53), Rule T: Condition Law</td>
</tr>
<tr>
<td>7</td>
<td>PVS</td>
<td>(\neg 346), Rule T: Resolution Law</td>
</tr>
</tbody>
</table>

\[ \text{PVS is a valid conclusion} \]

9. "If the labour market is perfect, then the wages of all persons in a particular employment are equal. But it is a case that wages for such persons are not equal. Therefore, the labour market is not perfect."

Test validity of the argument.

Let \( P \): Labour Market is Perfect

\( q \): Wages of all persons in a particular employment.

Premises: \( P \Rightarrow q \lor 17 \)

Conclusion: \( \neg P \)

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(P \Rightarrow q)</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>(17)</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>(\neg P)</td>
<td>(\neg 123), Rule T: Modus Tollens</td>
</tr>
</tbody>
</table>

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If there was a cricket, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore there was no cricket. Show that proposition is valid argument.

Sol: Let P: There was a cricket.
    q: Travelling was difficult.
    r: They arrived on time.

Premises: p → q, r → ¬q, r

Conclusion: ¬p.

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>r → ¬q</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>r</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>r → ¬q</td>
<td>$\neg q$</td>
</tr>
<tr>
<td>4</td>
<td>p → q</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>¬q</td>
<td>$\neg q$</td>
</tr>
</tbody>
</table>

The given proposition forms a valid conclusion.

Test the validity of the argument:

"If the music party could not play music or the refreshment were not delivered on time, then the new year’s party would have been cancelled, and the organiser Rammo would have been angry. If the party were cancelled, then refunds would have to be made. No refunds we made. Therefore music party could play music!"

Sol: Let P: If music party could play music.
    q: The refreshment were delivered on time.
    r: The new year party were cancelled.
    s: The organiser Rammo was angry.
    t: The funds has to be made.

Premises: (p ∧ ¬q) → (r ∧ s) ∧ t, ¬t

Conclusion: p.

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>r → t</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>r</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>r → ¬t</td>
<td>$\neg t$</td>
</tr>
<tr>
<td>4</td>
<td>r ∧ ¬q</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>r ∧ ¬q</td>
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<td>6</td>
<td>r ∧ ¬q</td>
<td>Rule P</td>
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<td>7</td>
<td>r ∧ ¬q</td>
<td>$\neg t$</td>
</tr>
<tr>
<td>8</td>
<td>r ∧ ¬q</td>
<td>Rule P</td>
</tr>
<tr>
<td>9</td>
<td>r ∧ ¬q</td>
<td>$\neg q$</td>
</tr>
</tbody>
</table>
1. INDIRECT METHOD

For doing indirect method introduce the negation of the conclusion as an additional premise and from the additional premise together with the given premise derive a contradiction.

1) Using indirect method of proof prove that $P \rightarrow R$, $\neg S, PVQ \rightarrow SVR$

Sol: Premises: $P \rightarrow R$, $\neg S, PVQ$

Conclusion: $SVR$ [Indirect Proof]

Additional Premise: $\neg (SVR) \rightarrow TS \land TR \land TP$

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$TS \land TR$</td>
<td>Additional Premises, Rule AP</td>
</tr>
<tr>
<td>2</td>
<td>$TS$</td>
<td>$\neg S$, Rule T: Simplification</td>
</tr>
<tr>
<td>3</td>
<td>$TR$</td>
<td>$P \rightarrow R$, Rule T: Simplification</td>
</tr>
<tr>
<td>4</td>
<td>$P \rightarrow R$</td>
<td>Rule T</td>
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<td>$TP$</td>
<td>$\neg S, PVQ$, Rule T: Modus Tollens</td>
</tr>
<tr>
<td>6</td>
<td>$PVQ$</td>
<td>Rule T</td>
</tr>
<tr>
<td>7</td>
<td>$\alpha$</td>
<td>$\alpha \rightarrow S$, Rule T: Disjunction</td>
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<td>8</td>
<td>$\alpha \rightarrow S$</td>
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</tr>
<tr>
<td>9</td>
<td>$S$</td>
<td>$\neg S, PVQ$, Rule T: Modus Ponens</td>
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<tr>
<td>10</td>
<td>$TS \land S$</td>
<td>$\neg S, PVQ$, Rule T: Conjunction</td>
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<tr>
<td></td>
<td>$\neg \neg R$</td>
<td>$\neg S, PVQ$, Rule T: Complement Law</td>
</tr>
</tbody>
</table>

$SVR$ is a valid conclusion.

2) Using indirect method show that $P \rightarrow q, q \rightarrow y, \neg (PVQ$, $\neg P$)

Sol: Premises: $P \rightarrow q, q \rightarrow y, \neg (PVQ$, $\neg P$)

Conclusion: $y$ Additional Premises: $\neg P$, Rule T
<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \neg \Psi )</td>
<td>Additional Premises; Rule AP; Rule ( \neg )</td>
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<td>2</td>
<td>( \Psi \land \Re )</td>
<td>Rule ( \land )</td>
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<tr>
<td>3</td>
<td>( \Re )</td>
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</tr>
<tr>
<td>4</td>
<td>( \Re )</td>
<td>Rule ( \neg )</td>
</tr>
<tr>
<td>5</td>
<td>( \Re \rightarrow \neg \Re )</td>
<td>Rule ( \neg \rightarrow )</td>
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<td>( \Re \rightarrow \neg \Re )</td>
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<td>( \neg \Re )</td>
<td>Rule ( \neg )</td>
</tr>
<tr>
<td>8</td>
<td>( \neg \Theta (\neg \Re) )</td>
<td>Rule ( \neg \neg )</td>
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<td>9</td>
<td>( \neg \Theta \neg \Re )</td>
<td>Rule ( \neg \neg )</td>
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<td>( \Re \rightarrow \neg \Re )</td>
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<tr>
<td>12</td>
<td>( \neg \Re \neg \Theta )</td>
<td>Rule ( \neg \neg )</td>
</tr>
<tr>
<td>13</td>
<td>( \neg \Theta )</td>
<td>Rule ( \neg )</td>
</tr>
</tbody>
</table>

\( \Theta \) is a valid conclusion.

Using indirect method of proof derive \( \Re \rightarrow \neg \Re \) from \( \Re \rightarrow (\neg \Re) \), \( \neg \Re \rightarrow \neg \Re \) and \( \neg \Re \).

Sol: Premises: \( \Re \rightarrow (\neg \Re) \), \( \neg \Re \rightarrow \neg \Re \), \( \neg \Re \).

Conclusion: \( \Re \rightarrow \neg \Re \).

Additional Premises: \( \neg \Theta (\neg \Re) \rightarrow \neg \Theta \neg \Re \) is valid.

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>13</td>
<td>( \Re )</td>
<td>Rule ( \neg )</td>
</tr>
</tbody>
</table>

\( \Re \rightarrow \neg \Re \) is a valid conclusion.
Show that the following implication by using indirect method

\[ R \Rightarrow \alpha, R V S, S \Rightarrow \alpha, P \Rightarrow \alpha \Rightarrow TP. \]

Sol. Premises: \( R \Rightarrow \alpha, R V S, S \Rightarrow \alpha, P \Rightarrow \alpha. \)

**Conclusion:** TP [Indirect Proof]

Additional Premises: \( \sim (TP) \equiv P \)

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASONS</th>
</tr>
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<td>4</td>
<td>( R \Rightarrow \alpha )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( S \Rightarrow \alpha )</td>
<td>Rule P</td>
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<td>6</td>
<td>( R V S \Rightarrow \alpha )</td>
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<td>Rule P</td>
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<td>( \alpha )</td>
<td>Rule P</td>
</tr>
<tr>
<td>9</td>
<td>( \alpha \land \neg \neg \alpha )</td>
<td>Rule P, Complementation Law</td>
</tr>
<tr>
<td>10</td>
<td>( \neg \alpha )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

\* TP is a valid Conclusion.

**Why**

1. Prove by indirect method \( \neg TP \Rightarrow \neg P \Rightarrow Q \Rightarrow P V R \Rightarrow \neg T \).
2. Using indirect method, show that \( P \Rightarrow \alpha, \alpha \Rightarrow R, P V R \Rightarrow \neg R \).
3. Use indirect method and prove that

   (a) \( R \Rightarrow \alpha, R V S, S \Rightarrow \alpha, P \Rightarrow \alpha \Rightarrow TP \).
   (b) \( P \Rightarrow \alpha, R \Rightarrow \alpha, S \Rightarrow (P V R), S \Rightarrow \alpha \).
   (c) \( E \Rightarrow S, S \Rightarrow H, S \Rightarrow \alpha \Rightarrow H \Rightarrow \neg (E \land A) \).

**CONDITIONAL PROOF**

If the conclusion is of the form \( P \Rightarrow Q \), then set

Additional Premise: \( P \)  
Conclusion: \( Q \)

1. Show that \( \neg Q \) can be derived from \( P \Rightarrow (\neg Q) \), \( R V P \Rightarrow \alpha \).

Sol. Premises: \( P \Rightarrow (\neg Q), R V P, \alpha - \)

Additional Premises: \( R \)  
Conclusion: \( \neg Q \)

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
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<tbody>
<tr>
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<td>Rule P</td>
</tr>
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<td>( R V P )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( P \Rightarrow (\neg Q) )</td>
<td>Rule P, Disjunction Law</td>
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<td>6</td>
<td>( S )</td>
<td>Rule P</td>
</tr>
<tr>
<td>7</td>
<td>( S \Rightarrow (P V R) )</td>
<td>Rule P</td>
</tr>
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<td>8</td>
<td>( S \Rightarrow (\neg Q) )</td>
<td>Rule P, Modus Ponens</td>
</tr>
<tr>
<td>9</td>
<td>( S )</td>
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</tr>
<tr>
<td>10</td>
<td>( S \Rightarrow (\neg Q) )</td>
<td>Rule P, Modus Ponens</td>
</tr>
</tbody>
</table>
4. Derive \( P \rightarrow (Q \rightarrow S) \) using the Rule CP from \( P \rightarrow (Q \rightarrow R) \), \( Q \rightarrow (R \rightarrow S) \)

Solution: Premises & \( P \rightarrow (Q \rightarrow R) \), \( Q \rightarrow (R \rightarrow S) \)

Conclusion: \( P \rightarrow (Q \rightarrow S) \) [Conditional Proof]

Additional Premises: \( P \)

Final Conclusion: \( Q \rightarrow S \)

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
</tr>
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<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>( Q \rightarrow R )</td>
<td>( 2.17 ), Rule T: Modus Tollens</td>
</tr>
<tr>
<td>3</td>
<td>( Q \rightarrow (R \rightarrow S) )</td>
<td>Rule P</td>
</tr>
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<tr>
<td>5</td>
<td>( V \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( T \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>7</td>
<td>( R \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>8</td>
<td>( S \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
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<tr>
<td>9</td>
<td>( Q \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
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<tr>
<td>10</td>
<td>( Q \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>11</td>
<td>( Q \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

Show that the hypothesis "If you send me an email message then I will finish writing the program." If you don't send me an email message then I will go to sleep early. If I go to sleep early then I will wake up feeling refreshed. Lead to the conclusion. If I don't finish writing the program then I will wake up feeling refreshed.

Solution: P: You send me an email message; Q: I will finish writing the program; R: I will go to sleep early; S: I will wake up feeling refreshed.

Premises: \( P \rightarrow Q \), \( Q \rightarrow R \), \( R \rightarrow S \)

Conclusion: \( T \rightarrow S \) [Conditional Proof]

Additional Premises: \( T \rightarrow Q \)

Final Conclusion: \( S \).

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( T \rightarrow R )</td>
<td>Rule AP</td>
</tr>
<tr>
<td>2</td>
<td>( P \rightarrow Q )</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>( T \rightarrow P )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( T \rightarrow R )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( T \rightarrow (Q \rightarrow S) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( Q \rightarrow S )</td>
<td>Rule P</td>
</tr>
<tr>
<td>7</td>
<td>( S )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

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1. Show that \( P \rightarrow \neg S \) can be derived from the premises \( P \rightarrow (L \rightarrow S) \lor R \lor T \lor P \rightarrow \neg S \).

2. Prove, using conditional proof:
   (a) \( P \rightarrow Q \rightarrow P \rightarrow (P \land Q) \).
   (b) \( (P \land Q) \rightarrow R \rightarrow (P \land Q) \rightarrow R \).
   (c) \( P \land P \rightarrow (Q \rightarrow R) \rightarrow S \rightarrow S \).

### Consistency and Inconsistency of Premises

A set of form \( H_1, H_2, \ldots, H_m \) is said to be

- **Inconsistent**: \( H_1 \land H_2 \land H_3 \land \ldots \land H_m \land \neg R \lor \neg T \lor E \)  
- **Consistent**: \( H_1 \land H_2 \land H_3 \land \ldots \land H_m \land E \lor V \lor T \lor E \)

1. Show that \( P \rightarrow Q \lor R \lor L \rightarrow T \) and \( P \) are inconsistent.

   **Solution:**
   - **Premises:** \( P \rightarrow Q \lor R \lor L \rightarrow T \) and \( P \)
   - **Reason:** \( \neg P \lor \neg Q \lor \neg R \lor \neg L \lor \neg T \lor \neg P \)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
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<tr>
<td>6</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
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<tr>
<td>7</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
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<td>8</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
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<td>9</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
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<tr>
<td>10</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
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<tr>
<td>11</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
<tr>
<td>12</td>
<td>( P \rightarrow Q \lor R \lor L \rightarrow T )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

- **Given set of premises are inconsistent.**

2. Prove that the premises \( a \rightarrow (b \rightarrow c) \), \( d \rightarrow (b \lor c) \), \( a \land d \) are inconsistent.

   **Solution:**
   - **Premises:** \( a \rightarrow (b \rightarrow c) \), \( d \rightarrow (b \lor c) \), \( a \land d \)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>7</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>8</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>9</td>
<td>( a \land d )</td>
<td>Rule P</td>
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<td>10</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>11</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
<tr>
<td>12</td>
<td>( a \land d )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

- **Given set of premises are inconsistent.**

3. Show that the following premises are inconsistent:
   1. If Jack misses many classes through illness, he fails HighSchool.
   2. If Jack fails high school, then he is uneducated.
   3. If Jack reads a lot of books, then he is not uneducated.
(iv) Jack Misses many class through illness & reads a lot of books.

So: Let P: Jack misses many classes through illness
      Q: Jack fails high school
      R: Jack is uneducated
      S: Jack reads a lot of books

Premises: P \rightarrow Q, Q \rightarrow R, S \rightarrow T \rightarrow R, \neg P, S.

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P \rightarrow Q</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>Q \rightarrow R</td>
<td>Rule P</td>
</tr>
<tr>
<td>3</td>
<td>P \rightarrow R</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>S \rightarrow T \rightarrow R</td>
<td>Rule T, Hypothetical Syllogism</td>
</tr>
<tr>
<td>5</td>
<td>T \rightarrow S</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>P \rightarrow T \rightarrow R</td>
<td>Rule T, Hypothetical Syllogism</td>
</tr>
<tr>
<td>7</td>
<td>T \rightarrow S</td>
<td>Rule T</td>
</tr>
<tr>
<td>8</td>
<td>T \rightarrow \neg P \rightarrow S</td>
<td>Rule T, Hypothetical Syllogism</td>
</tr>
<tr>
<td>9</td>
<td>T \rightarrow (P \land \neg P)</td>
<td>Rule T, Contradiction Law</td>
</tr>
<tr>
<td>10</td>
<td>(P \land \neg P) \rightarrow \neg P</td>
<td>Rule T, Conjunction Law</td>
</tr>
<tr>
<td>11</td>
<td>\neg P</td>
<td>Rule T, Completeness Law</td>
</tr>
</tbody>
</table>

Given set of premises are inconsistent.

(5) Show that the following premises are inconsistent.

(a) If Ram gets his degree he will go for a job. If he goes for a job, he will get married soon. If he goes for higher studies, he will not get married. Ram gets his degree and goes for higher studies.

(b) A diagnostic message is stored in a buffer or it is retransmitted. A diagnostic message is not stored in the buffer. If a diagnostic message is stored in the buffer then it is retransmitted. A diagnostic message is not transmitted.
**A. PREDICATE CALCULUS.**

Predicate: A part of a declarative sentence that attributes a property to the subject.

Example:
- Statement function: Proposition function, \( P(x) : x \) is a boy
- Proposition: \( P(\text{Raman}) : \text{Raman is a boy.} \)

<table>
<thead>
<tr>
<th>1-PLACE-PREDICATE</th>
<th>2-PLACE-PREDICATE</th>
<th>3-PLACE-PREDICATE</th>
<th>4-PLACE-PREDICATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) ): Ruthra is rich</td>
<td>( P(x, y) ): Ram is shorter than Ishy</td>
<td>( P(x, y, z) ): Rama wins by 1 point against James and John</td>
<td>( P(x, y, z) ): Jay &amp; Jill played bridge against James &amp; Raj</td>
</tr>
</tbody>
</table>

**PROPOSITIONAL FUNCTION: PROBLEMS**

1. Let \( P(x) \) denote the statement "\( x > 73 \)." What are the truth values of \( P(4) \) & \( P(2) \)?
   - \( P(4) = 73 \Rightarrow \text{false} \) & \( P(2) = 73 \Rightarrow \text{true} \)

2. Let \( \alpha (x, y) \) denote "\( x = y + 3 \)." What are the truth values of the proposition \( \alpha (1, 2) \) & \( \alpha (3, 10) \)?
   - \( \alpha (1, 2) : 1 + 2 + 3 = \text{false} \) & \( \alpha (3, 10) : 3 + 10 = \text{true} \)

3. Let \( R(x, y, z) \) denote "\( x + y = z \)." What are the truth values for the proposition \( R(1, 2, 3) \) & \( R(2, 1, 3) \)?
   - \( R(1, 2, 3) : 1 + 2 = 3 = \text{true} \) & \( R(2, 1, 3) : 2 + 1 = 3 = \text{true} \)

**SIMPLE STATEMENT FUNCTION**

Defined to be an expression consisting of a predicate symbol & an individual variable statement. When the variable is replaced by the name of any object.

Example:
- \( \text{Statement function: } \forall x : x \text{ is a teacher} \)
- \( \text{Statement: } (\text{John is a teacher}) \)

**STATEMENT FUNCTION & VARIABLES**

An expression consisting of a predicate symbol & 2 individual variables.

Example:
- \( \text{Statement function: } \forall x \forall y : x \text{ is taller than } y \)
- \( \text{Statement: } (\text{Ram is taller than John}) \)

**COMPOUND STATEMENT**

Obtained by combining one or more simple statement functions using logical connectives.

Example:
- \( \forall x : x \text{ is a man} \)
- \( H(x) : x \text{ is a mortal} \)

**QUANTIFIERS**

Quantifiers are one which is used to quantify the nature of variables such as "all, some, none or one".

**QUANTIFICATION**

To create a proposition from a statement function over a universe of discourse. The universe of discourse specifies the possible value of the variable.
**UNIVERSAL QUANTIFIERS**

The expression "All" is the Universal quantifier. We denote it by \( \forall x \).

The following phrases have the same meaning as "All":
1. For all \( x \)
2. For every \( x \)
3. For each \( x \)
4. Everything \( x \)
5. Each thing \( x \) is such that

**Examples:**
- \( \forall x \) \( x \) is a dog
- All dogs have a tail: \( \forall x \) [\( D(x) \rightarrow T(x) \)]
- No dog has a tail: \( \forall x \) [\( D(x) \rightarrow \neg T(x) \)]

**EXISTENTIAL QUANTIFIERS**

The expression "Some" is the Existential quantifier. We denote it by \( \exists x \).

The following phrases have the same meaning as "Some":
1. For some \( x \)
2. Some \( x \) is \( t \)
3. There exist \( x \) such that
4. There is a \( x \) such that
5. There is at least one \( x \) such that

**Examples:**
- \( \exists x \) \( x \) has a tail
- Some dog has a tail: \( \exists x \) [\( D(x) \land T(x) \)]
- Some dogs have a tail: \( \exists x \) [\( D(x) \land T(x) \)]
- No tail

**PROBLEMS UNDER UNIVERSE OF DISCOURSE**

1. Let \( P(x) \) be the statement "\( x + 1 > x \)." What is the truth value of the quantification \( \forall x \ P(x) \), where the universe of discourse consists of all real numbers?

**Sol:**
- Given: \( P(x) : x + 1 > x \)  
- Universe of discourse: All real numbers
- To find: Truth value: \( \forall x \ P(x) \)

\( P(x) \) is TRUE for all real no. \( x \cdot \cdot \cdot \) \( \forall x \ P(x) \) is TRUE.

2. Let \( Q(x) : x < 2 \). What is the truth value of the quantification \( \forall x \ Q(x) \), where the universe of discourse consists of all real no.?

**Sol:**
- Given: \( Q(x) : x < 2 \)  
- Universe of discourse: All real numbers
- To find: Truth value: \( \forall x \ Q(x) \)

\( Q(x) \) is not true for every real no. \( x \cdot \cdot \cdot \) \( Q(3) \) is false

3. Let \( R(x) : x > 3 \). What is the truth value of \( \exists x \ R(x) \), where the universe of discourse consists of all real numbers?

**Sol:**
- Given: \( R(x) : x > 3 \)  
- Universe of discourse: All real numbers
- To find: Truth value: \( \exists x \ R(x) \)

\( R(x) \) is true, when \( x = 4 \cdot \cdot \cdot \) \( \exists x \ R(x) \) is TRUE.

4. Let \( A = \{1, 2, 3, 4, 5, 6\} \). Determine the truth value of \( \forall x \ (x \in A) \)

**Sol:**
- Given: \( P(x) : x^2 > 25 \)  
- Universe of discourse: \( A = \{1, 2, 3, 4, 5, 6\} \)
- To find: Truth value: \( \forall x \ (x^2 > 25) \)

\( \exists x \ (x \in A) \ (x^2 > 25) \)

\( \forall x \ (x \in A) \ P(x) \) is TRUE since \( x = 6 \rightarrow 36 > 25 \)

5. Find the truth value of \( (\exists x) \ (P \rightarrow Q(x)) \lor (\exists x) \ R(x) \)

**Sol:**
- Given: \( P : x > 1 \)  
- **Universe of discourse:** \( E = \{2, 3, 4\} \)

\( \forall x : x > 3 \)  
(\( \exists x \ R(x) \) : \( x > 4 \))
To find Truth Value: \((\forall x)(P \rightarrow \alpha(x)) \lor (\exists x)R(x)\).

\(P\) is true and \(\alpha(1)\) is false.

\(\therefore (\exists x)(P \rightarrow \alpha(x))\) is false.

\((\forall x)R(x)\) is false [\(R(2), R(3), R(4)\) are all false].

\(\therefore (\exists x)(R(x))\) is false [\(\therefore \forall x \forall y \therefore F = F\)].

6. Let the universe of discourse be \(E = \{5, 6, 7\}.\) Let \(A = \{5, 6\}\) and \(B = \{6, 7\}.\) Let \(P(x)\): \(x\) is in \(A\) \(\exists \alpha(x)\): \(x\) is in \(B\) \(\subset R(x, y)\): \(x + y < 12\). Find the truth value of

\(\forall x (P(x) \rightarrow \exists \alpha(x)) \rightarrow R(5, 6)\).

\(\therefore \) Given \(P(x)\): \(x\) is in \(A\), \(A = \{5, 6\}\); \(\exists \alpha(x)\): \(x\) is in \(B\), \(B = \{6, 7\}\)

\(R(x, y)\): \(x + y < 12\)

To find Truth Value: \((\exists x)(P(x) \rightarrow \exists \alpha(x)) \rightarrow R(5, 6)\).

\(\therefore \) \(\forall x \rightarrow \exists \) \(\exists \alpha(x) \rightarrow R(5, 6)\) [\(5 + 6 < 12\)]

\(\therefore \) \(\therefore \forall x \rightarrow \exists \) \(\exists \alpha(x) \rightarrow R(5, 6)\) is true.

HW

6. Let \(P(x)\): \(x\) is an even integer \& \(R(x, y)\): \(x\) is divisible by \(y\). Let the universe of discourse be the set \(U = \{1, 2, 3, 4, 5, 6, 7\}\). Find the truth values of the following:

(a) \(P(1)\) \(\land P(4)\) \(\land R(4, 2)\) \(\land R(4, 4)\) \(\land R(4, 16)\)

2. Let \(P(x)\): \(x > 12\) and \(\alpha(x)\): \(x\) is a multiple of 10. Let the universe of discourse as all positive integers. Find the value of \((\exists x)(P(x) \rightarrow \alpha(x))\).

8. Let \(A = \{1, 2, 3, 4, 5\}\). Determine the truth value of each of the following:

(i) \(\forall x \in A (x^2 \leq 130)\)

(ii) \(\forall x \in A (x^2 = 30)\)

FREE AND BOUND VARIABLES

<table>
<thead>
<tr>
<th>BOUND VARIABLE</th>
<th>FREE VARIABLE</th>
<th>SCOPE OF VARIABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The variable is said to be bound if it is concerned with either universal ((\forall x)) or existential ((\exists x)) quantifier.</td>
<td>The variable which is not concerned with any quantifier is called free variable.</td>
<td>The scope of the quantifier is the formula immediately following the quantifier.</td>
</tr>
</tbody>
</table>

EXAMPLES:

Find the scope of the quantifiers and the nature of occurrence of the variable of the formula.

(i) \((\forall x) P(x, y)\): Bound variable: \(x\) Free variable: \(y\)

(ii) \((\exists x) \rho(x, y)\): Bound variable: \(x\) Free variable: \(y\)
Examples: Negate the following statements.

(a) P: Every student in your class has taken a course in Calculus.


(b) P: No one has done every problem in the exercise.


(c) P: Some people who trust others are rewarded.


(d) P: To enter into the country you need a passport or a voter registration card.


Translate the following into logical expressions-symbolic.

(e) Every student in this class has studied Calculus.


(f) Restatement: For every person x, if x is a student in this class then x has studied Calculus.


(g) Proposition: s(x): x is a student in this class.


(h) Function: c(x): x has studied calculus.


(i) Symbolic form: ∀x [s(x) → c(x)].
Some students in the class has visited Mexico.
Every student in this class has visited either Canada or Mexico.

Sol: Restatement:

1. For some \( x \), \( x \) is a student in this class and has visited Mexico.
2. For every \( x \), if \( x \) is a student in this class then \( x \) has visited either Canada or Mexico.

Proposition function:
\( S(x) : x \) is a student in the class
\( M(x) : x \) has visited Mexico
\( C(x) : x \) has visited Canada.

Symbolic form:
1. \( \exists x (S(x) \land M(x)) \)
2. \( \forall x (S(x) \rightarrow (C(x) \lor M(x))) \)

3. All lions are fierce
4. Some lions do not drink coffee.
5. Some fierce creatures do not drink coffee.

Sol: Restatement:

1. For every \( x \), if \( x \) is a lion then \( x \) is fierce.
2. For some \( x \), \( x \) is a lion and does not drink coffee.
3. For some \( x \), \( x \) is a fierce creature and does not drink coffee.

Proposition function:
\( L(x) : x \) is a lion
\( F(x) : x \) is a fierce creature

Symbolic form:
1. \( \forall x (L(x) \rightarrow F(x)) \)
2. \( \exists x (L(x) \land \lnot C(x)) \)
3. \( \exists x (F(x) \land \lnot C(x)) \)

4. Let \( (x, y) : x \) is taller than \( y \).
   Translate the following into a formula:
   for any \( x \) and \( y \), \( y \) is taller than \( y \) then it is not true that \( y \) is taller than \( x \).

Sol: Proposition function:
\( T(x, y) : x \) is taller than \( y \)
\( L(x, y) : y \) is taller than \( x \).

Symbolic form:
\( T(x, y) \rightarrow \neg L(y, x) \)

Nested quantifier: Two quantifiers are nested if one is within the scope of the other. E.g.: \( \forall x \exists y (x + y = 0) \)

5. All the world loves a lover.

Sol: Restatement:
For all \( x, y : x \) is a person then for all \( y \) if \( y \) is a person and \( y \) is a lover then \( x \) loves \( y \).

Proposition function:
\( P(x) : x \) is a person
\( L(x) : x \) is a lover
\( R(x, y) : x \) loves \( y \).

Symbolic form:
\( \forall x (P(x) \rightarrow \forall y (y \in P \land L(y) \rightarrow R(x, y))) \)
6. It is not true that all roads lead to Denmark.

Restatement: It is false that for all $x$, if $x$ is a road then $x$ leads to Denmark.

Proposition: $P(x) : x$ is a road

Function: $D(x) : x$ leads to Denmark.

Symbolic form: $\forall x (P(x) \rightarrow D(x))$

7. No one has done every problem in the exercise.

Restatement: The statement also.

Solution: $P: \neg \forall x \exists y (D(x,y))$

Negation: $\neg P : \exists x \forall y \neg (D(x,y))$

8. Write the symbolic form of $x$ is the father of the mother of $y$.

(i) Given any positive integer, there is a greater positive integer.

With & Without using set of positive integers as the universe of discourse.

Solution: (i) Statement: $x$ is the father of the mother of $y$.

Restatement: let $z$ as mother of $y$. Then, there is a $z$ such $x$ is the father of $z$ and $z$ the mother of $y$.

Proposition: $P(x) : x$ is a person.

$F(x,y) : x$ is the father of $y$.

$M(x,y) : x$ is the mother of $y$.

Symbolic form: $\exists z (P(z) \land F(z,y) \land M(z,y))$

(ii) Statements: Given any positive integer, there is a greater positive integer.

(iii) Universe of discourse: Positive Integer.

Restatement: if $x$ & $y$ are positive integers then "for all $x$ there is $y$ is greater.

Proposition: $G(x,y) : x$ is greater than $y$.

Symbolic form: $\forall x \exists y (G(x,y))$

(iv) Universe of discourse: Without using positive integers.

Proposition: $P(x) : x$ is positive.

$G(x,y) : x$ is greater than $y$.

Symbolic form: $\forall x (P(x) \rightarrow \exists y (P(y) \land G(x,y)))$
Translate the statement to symbolic form.

1. The sum of two positive integers is always positive.
   \[ x + y (x > 0) \land (y > 0) \implies (x + y > 0) \]
2. Every real number except zero has a multiplication inverse.
   \[ x (x \neq 0) \implies \exists y (x y = 1) \]
3. The product of a positive real number and a negative real number is always a negative real number.
   \[ x y (x > 0) \land (y < 0) \implies (x y < 0) \]
4. If a person is female and is a parent then the person is someone's mother.
   \[ x [\text{F}(x) \land \text{P}(x)] \implies \exists y \text{M}(x, y) \]
5. If any one is good then John is good.
   \[ \forall x [x \text{ good}] \implies \text{John is good} \]
6. If there is one x such that x is a person and x is good then John is good.
   \[ \exists x [\text{P}(x) \land x \text{ is good}] \implies \text{John is good} \]
7. If he is ambitious or no one is ambitious.
   \[ \text{H} \text{ is ambitious} \lor \neg \forall x \text{P}(x) \]
8. If he is ambitious or for all x, if x is a person then x is not bad.
   \[ \text{H} \text{ is ambitious} \lor \forall x [\text{P}(x) \implies \neg \text{bad}(x)] \]
6. Everyone who likes fun will enjoy each of these play.
   Sol: \( L(x) : x \text{ likes fun} \); \( P(x) : x \text{ is a play} \); \( E(y) : y \text{ will enjoy} \)
   \[ \forall x [L(x) \rightarrow \exists y (P(y) \rightarrow E(x, y))]. \]

<table>
<thead>
<tr>
<th>2M X RULES OF INFERRENCE FOR QUANTIFIED STATEMENT</th>
</tr>
</thead>
</table>
| 1. UNIVERSAL SPECIFICATION : \( \{ x \} \) \[ \forall x \; P(x) \]
   \[ \therefore \; P(c) \]
   \[ \forall x (P(x) \rightarrow P(c)); \text{ for some } c \]
| 2. UNIVERSAL GENERALISATION : \( \{ x \} \) \[ P(c) \]
   \[ \therefore \; \forall x \; P(x) \]
   \[ P(c) \Rightarrow \forall x P(x); \text{ for any arbitrary } c \]
| 3. EXISTENTIAL SPECIFICATION \( \{ x \} \) \[ \exists x \; P(x) \]
   \[ \forall x (P(x) \rightarrow P(c)) \]
   \[ \exists x \; P(x) \Rightarrow P(c); \text{ for particular } c \]
| 4. EXISTENTIAL GENERALISATION \( \{ x \} \) \[ \exists x \; P(x) \]
   \[ \forall x (P(x) \rightarrow P(c)) \]
   \[ \exists x \; P(x) \Rightarrow \exists x \; P(x) \]

<table>
<thead>
<tr>
<th>LOGICAL EQUIVALENCE AND IMPLICATIONS FOR QUANTIFIED STATEMENT IN ONE VARIABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \exists x [P(x) \land q(x)] ) ( \iff ) ( \exists x ; P(x) \land \exists x ; q(x) )</td>
</tr>
<tr>
<td>2. ( \forall x [P(x) \lor q(x)] ) ( \iff ) ( (\forall x) P(x) \lor (\forall x) q(x) )</td>
</tr>
<tr>
<td>3. ( \forall x [P(x) \land q(x)] ) ( \iff ) ( (\forall x) P(x) \land (\forall x) q(x) )</td>
</tr>
<tr>
<td>4. ( \forall x P(x) \lor \forall x q(x) ) ( \iff ) ( (\forall x) [P(x) \lor q(x)] )</td>
</tr>
<tr>
<td>5. ( \forall x [P(x) \land q(x) \land r(x)] ) ( \iff ) ( (\forall x) [P(x) \land q(x)] \land r(x) )</td>
</tr>
<tr>
<td>6. ( \exists x [P(x) \Rightarrow q(x)] ) ( \iff ) ( \exists x ; [\neg P(x) \lor q(x)] )</td>
</tr>
<tr>
<td>7. ( \forall x ; \neg (P(x) \land r(x)) ) ( \iff ) ( \forall x ; [\neg P(x) \lor \neg r(x)] )</td>
</tr>
<tr>
<td>8. ( \forall x ; \neg (P(x) \lor q(x)) ) ( \iff ) ( \forall x ; [\neg P(x) \land \neg q(x)] )</td>
</tr>
<tr>
<td>9. ( \forall x ; \neg [P(x) \Rightarrow q(x)] ) ( \iff ) ( [\forall x ; P(x) \Rightarrow \neg q(x)] )</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>PROBLEMS UNDER QUANTIFIED STATEMENT : 8M X0 X0</th>
</tr>
</thead>
</table>
| 1. Show that \( (\exists x) M(x) \) follows logically from premises \( \{ x \} \; (H(x) \rightarrow M(x)) \) \& \( (\exists x) \; H(x) \)
   Sol: Premises : \( \{ x \} \; (H(x) \rightarrow M(x)) \) \& \( (\exists x) \; H(x) \)
   Conclusion : \( \exists x \; M(x) \) \[ \text{ [DIRECT PROOF] \] |

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### Steps 1-6: Premises and Reasoning

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C \Rightarrow (H(C) \Rightarrow M(C)) \Rightarrow (\forall x)(H(x) \Rightarrow M(x))$</td>
<td>Rule P.</td>
</tr>
<tr>
<td>2</td>
<td>H(C) \Rightarrow M(C)</td>
<td>Rule us.</td>
</tr>
<tr>
<td>3</td>
<td>$\forall x \ H(x)$</td>
<td>Rule P.</td>
</tr>
<tr>
<td>4</td>
<td>H(C)</td>
<td>Rule Es.</td>
</tr>
<tr>
<td>5</td>
<td>M(C)</td>
<td>Rule Eq.</td>
</tr>
<tr>
<td>6</td>
<td>$\forall x \ M(x)$</td>
<td>Rule: Modus Ponens</td>
</tr>
</tbody>
</table>

**Prove that:**

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\forall x \ (P(x) \Rightarrow Q(x)) \land \forall x \ R(x) \Rightarrow \forall x \ (Q(x) \Rightarrow \forall x \ O(x))$</td>
<td>Rule: Direct Proof</td>
</tr>
</tbody>
</table>

**Solution:**

- Premises: $\forall x \ (P(x) \Rightarrow Q(x))$, $\forall x \ R(x) \Rightarrow \forall x \ O(x)$
- Conclusion: $\forall x \ (Q(x) \Rightarrow \forall x \ O(x))$

### Steps 7-8: Premises and Reasoning

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\forall x \ (P(x) \Rightarrow Q(x)) \Rightarrow \forall x \ P(x) \Rightarrow Q(x)$</td>
<td>Rule P.</td>
</tr>
<tr>
<td>2</td>
<td>PC(C) \Rightarrow QC(C)</td>
<td>Rule us.</td>
</tr>
<tr>
<td>3</td>
<td>$\forall x \ (R(x) \Rightarrow (Q(x) \Rightarrow T(x)))$</td>
<td>Rule P.</td>
</tr>
<tr>
<td>4</td>
<td>RC(C) \Rightarrow QC(C)</td>
<td>Rule us.</td>
</tr>
<tr>
<td>5</td>
<td>QC(C) \Rightarrow TR(C)</td>
<td>Rule Eq.</td>
</tr>
<tr>
<td>6</td>
<td>RC(C) \Rightarrow TR(C)</td>
<td>Rule: Conjunction</td>
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<tr>
<td>7</td>
<td>RC(C) \Rightarrow TR(C)</td>
<td>Rule: Hypothetical Syllogism</td>
</tr>
<tr>
<td>8</td>
<td>$\forall x \ CR(x) \Rightarrow TR(P(x))$</td>
<td>Rule: Direct Proof</td>
</tr>
</tbody>
</table>

### Steps 9-10: Premises and Reasoning

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\forall x \ (P(x) \land Q(x)) \Rightarrow \forall x \ Q(x)$</td>
<td>Rule: Direct Proof</td>
</tr>
<tr>
<td>2</td>
<td>$\forall x \ P(x) \land Q(x)$</td>
<td>Rule: US.</td>
</tr>
</tbody>
</table>

**Solution:**

- Premises: $\forall x \ (P(x) \land Q(x))$
- Conclusion: $\forall x \ Q(x)$

### Steps 11-12: Premises and Reasoning

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\forall x \ (P(x) \land Q(x)) \Rightarrow \forall x \ (P(x) \land Q(x))$</td>
<td>Rule: Direct Proof</td>
</tr>
<tr>
<td>2</td>
<td>PC(C) \land QC(C)</td>
<td>Rule: US.</td>
</tr>
</tbody>
</table>

**Solution:**

- Premises: $\forall x \ (P(x) \land Q(x))$
- Conclusion: $\forall x \ (P(x) \land Q(x))$
5. Show that \( \forall x \ (P(x) \Rightarrow q(x)) \land \forall x \ (P(x) \land s(x)) \Rightarrow \exists x \ (y(x) \land s(x)) \).

**Solution:**
1. Premises: \( \forall x \ (P(x) \Rightarrow q(x) \land s(x)) \), \( \exists x \ (P(x) \land s(x)) \).
   
   **Conclusion:** \( \exists x \ [y(x) \land s(x)] \).  
   
   **Direct Proof.**

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \forall x \ (P(x) \Rightarrow q(x) \land s(x)) )</td>
</tr>
<tr>
<td>2</td>
<td>( P(c) \Rightarrow q(c) \land s(c) )</td>
</tr>
<tr>
<td>3</td>
<td>( \exists x \ (P(x) \land s(x)) )</td>
</tr>
<tr>
<td>4</td>
<td>( P(c) \land s(c) )</td>
</tr>
<tr>
<td>5</td>
<td>( q(c) \land s(c) )</td>
</tr>
<tr>
<td>6</td>
<td>( q(c) )</td>
</tr>
<tr>
<td>7</td>
<td>( y(c) \land s(c) )</td>
</tr>
<tr>
<td>8</td>
<td>( y(c) )</td>
</tr>
<tr>
<td>9</td>
<td>( y(c) \land s(c) )</td>
</tr>
<tr>
<td>10</td>
<td>( \exists x \ [y(x) \land s(x)] )</td>
</tr>
</tbody>
</table>

6. Show that \( T(a,b) \) follows logically from \( T(\omega(a),b) \) and \( \forall x(y) (P(x,y) \Rightarrow \omega(x,y)) \).

**Solution:**
1. Premises: \( (x)(y) (P(x,y) \Rightarrow \omega(x,y)) \), \( T(\omega(a),b) \).
   
   **Conclusion:** \( T(a,b) \).  
   
   **Direct Proof.**

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \omega(a,y) \Rightarrow \omega(x,y) )</td>
</tr>
<tr>
<td>2</td>
<td>( \omega(x,y) \Rightarrow \omega(a,y) )</td>
</tr>
<tr>
<td>3</td>
<td>( P(a,b) \Rightarrow \omega(a,b) )</td>
</tr>
<tr>
<td>4</td>
<td>( T(\omega(a),b) )</td>
</tr>
<tr>
<td>5</td>
<td>( T(a,b) )</td>
</tr>
</tbody>
</table>

In nested quantifiers, leftmost quantifier is to be applied first then the next.

7. Show that \( (\exists x) (F(x) \Rightarrow \exists y \omega(x,y)) \) follows logically from

(a) \( (\exists x) (F(x) \land s(x)) \Rightarrow (y) (M(y) \Rightarrow \omega(y)) \)

(b) \( (y) (M(y) \Rightarrow \omega(y)) \)
Sol: Premises: \( \exists x \left( F(x) \land x = x \right) \implies (y)(M(y) \implies w(y)) \), \( \exists y (M(y) \land w(y)) \)

Conclusion: \( \exists x (F(x) \implies \neg (x)) \)  \[ \text{Direct Proof} \]

<table>
<thead>
<tr>
<th>STEPS</th>
<th>PREMISES</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \exists y \left( M(y) \land w(y) \right) )</td>
<td>Rule E.E.</td>
</tr>
<tr>
<td>2</td>
<td>( M(c) \land w(c) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>3</td>
<td>( \neg \exists y \left( M(y) \land w(y) \right) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>4</td>
<td>( \neg (M(c) \land w(c)) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>5</td>
<td>( \neg \exists y \left( M(y) \land w(y) \right) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>6</td>
<td>( \neg \exists y \left( M(y) \land w(y) \right) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>7</td>
<td>( \exists F(x) \land x = x ) ( \implies (y)(M(y) \implies w(y)) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>8</td>
<td>( \exists F(x) \land x = x ) ( \implies (y)(M(y) \implies w(y)) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>9</td>
<td>( x ) ( \exists F(x) \land x = x ) ( \implies (y)(M(y) \implies w(y)) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>10</td>
<td>( \exists F(x) \land x = x ) ( \implies \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>11</td>
<td>( \exists F(x) \land x = x ) ( \implies \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>12</td>
<td>( \exists F(x) \land x = x ) ( \implies \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>13</td>
<td>( \exists F(x) \land x = x ) ( \implies \neg (x) )</td>
<td>Rule T.</td>
</tr>
</tbody>
</table>

\[ \therefore \exists F(x) \land x = x \] is a valid conclusion.

3) Use Indirect Method of Proof to prove that:

\( \therefore \neg (x) (P(x) \lor \neg (x)) \implies \neg (x) \neg x \).

Sol: Premises: \( (x) (P(x) \lor \neg (x)) \)

Conclusion: \( (x) P(x) \lor \neg (x) \lor \neg (x) \)  \[ \text{Indirect Proof} \]

Additional Premises: \( \neg [(x) P(x) \lor \neg (x)] = \exists x \neg P(x) \land \neg (x) \lor \neg (x) \).

<table>
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<tbody>
<tr>
<td>1</td>
<td>( (x) \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule AP</td>
</tr>
<tr>
<td>2</td>
<td>( (x) \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>3</td>
<td>( (x) \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>4</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>5</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>6</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>7</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>8</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>9</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>10</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
<tr>
<td>11</td>
<td>( \neg P(x) \land \neg (x) \lor \neg (x) )</td>
<td>Rule T.</td>
</tr>
</tbody>
</table>

\[ \therefore (x) P(x) \lor \neg (x) \lor \neg (x) \] is a valid conclusion.

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3. Use of Rule and obtain the following implication

\[(x) \left( p(x) \rightarrow q(x) \right), (x) \left( r(x) \rightarrow \neg q(x) \right) \Rightarrow (x) \left( r(x) \rightarrow \neg p(x) \right) \]

**Sol.: Premises:** \((x) \left( p(x) \rightarrow q(x) \right), (x) \left( r(x) \rightarrow \neg q(x) \right)\)

**Conclusion:** \((x) \left( r(x) \rightarrow \neg p(x) \right)\) \[\text{Rule Conditional Proof}\]

**Additional Premises:** \((x) \ r(x)\)

**Final Conclusion:** \((x) \ r(x)\)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (x) \ r(x) )</td>
<td>Rule AP.</td>
</tr>
<tr>
<td>2</td>
<td>( r(a) )</td>
<td>Rule US.</td>
</tr>
<tr>
<td>3</td>
<td>( (x) \left( p(x) \rightarrow q(x) \right) )</td>
<td>Rule Ep.</td>
</tr>
<tr>
<td>4</td>
<td>( p(a) \rightarrow q(a) )</td>
<td>Rule US.</td>
</tr>
<tr>
<td>5</td>
<td>( (x) \left( r(x) \rightarrow \neg q(x) \right) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( r(a) \rightarrow \neg q(a) )</td>
<td>Rule US.</td>
</tr>
<tr>
<td>7</td>
<td>( \neg q(a) )</td>
<td>Rule US.</td>
</tr>
<tr>
<td>8</td>
<td>( \neg p(a) )</td>
<td>Rule US.</td>
</tr>
<tr>
<td>9</td>
<td>( (x) \left( \neg p(x) \right) )</td>
<td>Rule US.</td>
</tr>
</tbody>
</table>

\( (x) \left( r(x) \rightarrow \neg p(x) \right) \) is a valid conclusion.

4. Show that the Premises "Everyone in this discrete mathematics class has taken a course in Computer Science" and "Mala is a student in this class", imply the conclusion "Mala has taken a course in CS".

**Sol.: Proposition:** \( p(x) : x \text{ is in this discrete mathematics class} \), \( p(c) : x \text{ is a student in this class} \), \( p(x) : x \text{ has taken a course in Computer Science} \), \( p(c) : \text{Mala has taken a course in Computer Science} \)

**Premises:** \( \forall x \ (p(x) \rightarrow p(c)) \), \( p(c) \)

**Conclusion:** \( p(c) \)

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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \forall x \left( p(x) \rightarrow p(c) \right) )</td>
<td>Rule Ep.</td>
</tr>
<tr>
<td>2</td>
<td>( p(c) \rightarrow p(c) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>3</td>
<td>( p(c) \rightarrow p(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( p(c) \rightarrow p(c) )</td>
<td>Rule US</td>
</tr>
</tbody>
</table>

5. Show that the Premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".

**Sol.: Proposition:** \( p(x) : x \text{ is a student in this class} \), \( q(x) : x \text{ has read the book} \)

<table>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>( \forall x \left( p(x) \rightarrow q(x) \right) \Rightarrow \forall x \left( q(x) \rightarrow p(x) \right) )</td>
<td>Rule Ep.</td>
</tr>
<tr>
<td>2</td>
<td>( p(a) \rightarrow q(a) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>3</td>
<td>( q(a) \rightarrow p(a) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( p(a) \rightarrow q(a) )</td>
<td>Rule US</td>
</tr>
</tbody>
</table>

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(12) Show that the following premises are valid. All integers are rational numbers. Some integers are power of 2. Therefore, some rational number are power of 2.

Sol: Proposition
- \( p(x) \): \( x \) is an integer.
- \( q(x) \): \( x \) is a rational number.
- \( r(x) \): \( x \) is a power of 2.

Premises: \( \exists x (p(x) \land q(x)) \), \( \forall x (p(x) \rightarrow r(x)) \).

Conclusion: \( \exists x (q(x) \land r(x)) \).

<table>
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<tr>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>( \exists x (p(x) \land q(x)) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>( p(c) \land q(c) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>3</td>
<td>( p(c) \rightarrow r(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>( \exists x (p(x) \land r(x)) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( p(c) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>6</td>
<td>( r(c) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>7</td>
<td>( q(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>8</td>
<td>( q(c) \land r(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>9</td>
<td>( \exists x [q(x) \land r(x)] )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>

(13) Show that the following argument is valid. "Every microcomputer has a serial interface Port." "Some microcomputers have a parallel Port." Therefore, some microcomputers have both serial and parallel Ports.

Sol: Proposition: \( p(x) \): \( x \) is a microcomputer.
- \( q(x) \): \( x \) has a serial interface port.
- \( r(x) \): \( x \) has a parallel port.

Premises: \( \forall x (p(x) \rightarrow q(x)) \), \( \exists x (p(x) \land r(x)) \).

Conclusion: \( \exists x (q(x) \land r(x)) \).

<table>
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<tr>
<th>STEPS</th>
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<th>CONCLUSION</th>
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<tr>
<td>1</td>
<td>( \forall x (p(x) \rightarrow q(x)) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>( \exists x (p(x) \land q(x)) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>3</td>
<td>( p(c) )</td>
<td>Rule US</td>
</tr>
<tr>
<td>4</td>
<td>( q(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>5</td>
<td>( q(c) \land r(c) )</td>
<td>Rule P</td>
</tr>
<tr>
<td>6</td>
<td>( \exists x [q(x) \land r(x)] )</td>
<td>Rule P</td>
</tr>
</tbody>
</table>
**Premises:** 
\[ \forall x \ (P(x) \rightarrow q(x)) \land \exists x \ (P(x) \land q(x)) \]

**Conclusion:** 
\[ \exists x \ (P(x) \land q(x) \land q(x)) \]

<table>
<thead>
<tr>
<th>Steps</th>
<th>PreMises</th>
<th>Reason</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>(\forall x \ (P(x) \rightarrow q(x)))</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>(P(c) \rightarrow q(c))</td>
<td>(\text{S13, Rule US})</td>
</tr>
<tr>
<td>3</td>
<td>(\exists x \ (P(x) \land q(x)))</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>(P(c) \land q(c))</td>
<td>(\text{S34, Rule ES})</td>
</tr>
<tr>
<td>5</td>
<td>(q(c))</td>
<td>(\text{S43, Rule T: Simplification})</td>
</tr>
<tr>
<td>6</td>
<td>(q(c))</td>
<td>(\text{S43, Rule T: Simplification})</td>
</tr>
<tr>
<td>7</td>
<td>(q(c) \land q(c))</td>
<td>(\text{S2} \rightarrow \text{S2, Rule T: Simplification})</td>
</tr>
<tr>
<td>8</td>
<td>(P(c) \land [q(c) \land q(c)])</td>
<td>(\text{S7} \rightarrow \text{S7, Rule T: Conjunction})</td>
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<tr>
<td>9</td>
<td>(P(c) \land [q(c) \land q(c)])</td>
<td>(\text{S8} \rightarrow \text{S8, Rule T: Conjunction})</td>
</tr>
<tr>
<td>10</td>
<td>(\exists x \ (P(x) \land q(x) \land q(x)))</td>
<td>(\text{S9} \rightarrow \text{S9, Rule EQ})</td>
</tr>
</tbody>
</table>

10. Test the validity of the following argument:

If an integer is divisible by 10, then it is divisible by 2.
If an integer is divisible by 2, then it is divisible by 3.

The integer divisible by 10 is also divisible by 3.

**Proposition:** \(D_1(x): x\) is divisible by 10.
\(D_2(x): x\) is divisible by 2.
\(D_3(x): x\) is divisible by 3.

**Premises:** \((\forall x) \ (D_1(x) \rightarrow D_2(x))\), \((\forall x) \ (D_2(x) \rightarrow D_3(x))\)

**Conclusion:** \(\forall x \ (D_1(x) \rightarrow D_3(x))\)

<table>
<thead>
<tr>
<th>Steps</th>
<th>PreMises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\forall x \ (D_1(x) \rightarrow D_2(x)))</td>
<td>Rule P</td>
</tr>
<tr>
<td>2</td>
<td>(D_1(a) \rightarrow D_2(a))</td>
<td>(\text{S13, Rule US})</td>
</tr>
<tr>
<td>3</td>
<td>(\forall x \ (D_2(x) \rightarrow D_3(x)))</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>(D_2(a) \rightarrow D_3(a))</td>
<td>(\text{S33, Rule US})</td>
</tr>
<tr>
<td>5</td>
<td>(D_1(a) \rightarrow D_3(a))</td>
<td>(\text{S2,13, Rule T: Hypothetical Syllow})</td>
</tr>
<tr>
<td>6</td>
<td>((\forall x) \ (D_1(x) \rightarrow D_3(x)))</td>
<td>(\text{S5,3, Rule EQ})</td>
</tr>
</tbody>
</table>

13. Show that the premises "One student in this class knows how to write programs in JAVA" and "Everyone knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in class can get a high paying job".

**Proposition:** \(A(x): x\) is in the class.
\(B(x): x\) knows JAVA programing.
\(H(x): x\) can get a high paying job.

**Premises:** \(\exists x \ (A(x) \land B(x))\), \(\forall x \ (B(x) \rightarrow H(x))\)

**Conclusion:** \(\exists x \ (A(x) \land B(x) \land H(x))\)

<table>
<thead>
<tr>
<th>Steps</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\exists x \ (A(x) \land B(x)))</td>
<td>Rule P</td>
</tr>
<tr>
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<td>(A(c) \land B(c))</td>
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</tr>
<tr>
<td>3</td>
<td>(B(c) \rightarrow H(c))</td>
<td>Rule P</td>
</tr>
<tr>
<td>4</td>
<td>(A(c) \land B(c) \land H(c))</td>
<td>(\text{S2,13, Rule T: Hypothetical Syllow})</td>
</tr>
</tbody>
</table>
# Problem Statement

Verify that Validating of the following inference. If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than Siva. Therefore, Ram is not more successful than Siva.

**Proposition:**  
sc(xy) : x is more successful than y,  
H(xy) : x has worked harder than y  
To deserve success,

- a : Ram, b : Siva

**Premises:**  
csc(xy), [sc(xy) ⊃ H(xy)] (1), [H(xy) ⊃ H(xy)] (2),  
sc(a,b) (3), H(a,b) (4),  
H(a,b) (5)

**Conclusion:**  
T(sc(a,b))

# Table

<table>
<thead>
<tr>
<th>Steps</th>
<th>Premises</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>sc(xy), [sc(xy) ⊃ H(xy)]</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>[sc(xy) ⊃ H(xy)]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>sc(a,b) ⊃ H(a,b)</td>
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</tr>
<tr>
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<td>H(a,b)</td>
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<tr>
<td>5</td>
<td>H(a,b)</td>
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</tr>
<tr>
<td></td>
<td>[sc(a,b) ⊃ H(a,b)]</td>
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<tr>
<td></td>
<td>H(a,b)</td>
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</tbody>
</table>

**Reasons:**  
- Rule P.  
- Rule US.  
- Rule US.  
- Rule P.  
- Rule US.
**INTRODUCTION TO PROOFS**

**THEOREM:** A theorem is a statement that can be shown to be true.

**PROOF:** A proof is a valid argument that establishes the truth of a mathematical statement.

---

### Direct Proof

A direct proof shows that a conditional statement $P \Rightarrow Q$ is true by showing that if $P$ is true then $Q$ must be true.

### Proof by Contradiction

In proof by contradiction, we assume that $P \Rightarrow Q$ is false. If we can derive a contradiction, then by rules of inference, we conclude that $P \Rightarrow Q$ is false. Thus, indirectly, we prove $P \Rightarrow Q$.

### Vacuous Proof

The vacuous proof of $P \Rightarrow Q$ is to show that $P$ is false.

### Trivial Proof

The trivial proof of $P \Rightarrow Q$ is to show that $Q$ is true.

### Proofs of Equivalence

To prove a theorem of the form $P \iff Q$, we show that $P \Rightarrow Q$ and $Q \Rightarrow P$ are both true.

### Counter-Examples

A statement of the form $\forall x P(x)$ is false if we need only to find a counter-example that is an example $x$ for which $P(x)$ is false.

---

### SUMMARY OF PROOFS

<table>
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<tr>
<td><strong>Direct Proof</strong></td>
<td>$P \Rightarrow Q = \text{TRUE}$</td>
<td>$P \ \text{TRUE}$</td>
<td>$Q \ \text{TRUE}$</td>
</tr>
<tr>
<td><strong>Indirect Proof</strong></td>
<td>$P \Rightarrow Q = \text{TRUE}$</td>
<td>$T \ \text{TRUE}$</td>
<td>$T \ \text{TRUE}$</td>
</tr>
<tr>
<td><strong>Vacuous Proof</strong></td>
<td>$P \Rightarrow Q = \text{FALSE}$</td>
<td>$Q \ \text{FALSE}$</td>
<td>$P \ \text{FALSE}$</td>
</tr>
<tr>
<td><strong>Trivial Proof</strong></td>
<td>$P \Rightarrow Q = \text{TRUE}$</td>
<td>$Q \ \text{TRUE}$</td>
<td></td>
</tr>
<tr>
<td><strong>Proof of Equivalence</strong></td>
<td>$P \Rightarrow Q = \text{TRUE}$</td>
<td>$Q \Rightarrow P$</td>
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<tr>
<td><strong>Counter Example</strong></td>
<td>$\forall x P(x) = \text{FALSE}$</td>
<td>$\exists x P(x)$</td>
<td></td>
</tr>
</tbody>
</table>
TYPE 1: PROBLEMS BASED ON DIRECT PROOFS.

6. Give a direct proof of the statement: "The square of an odd integer is an odd integer".

Sol: Given: "The square of an odd integer is an odd integer" (or)
"If n is an odd integer, then n² is an odd integer".

Theorem: ∀n (P(n) → Q(n)), where P(n): n is an odd integer.

Proof of Direct Proof: P → Q is true.

Hypothesis: Assume that P is true i.e. n is an odd integer.

Analysis: By definition of an odd integer, n = 2k + 1, for some integer k.

n² = (2k + 1)² = 4k² + 4k + 1 = 2(2k² + 2k) + 1

Conclusion: R.H.S. = 2(2k² + 2k) + 1 is not divisible by 2.

→ n² is not divisible by 2.

→ n² is an odd integer.

∴ P → Q is true.

Hence, if n is an odd integer, then n² is an odd integer.

7. Give a direct proof of "The sum of two odd integers is even".

Sol: Given: If n is odd and m is odd then n + m is an even integer.

Theorem: ∀n [P(n) → Q(n)], where P(n): n is odd integer & m is odd integer.

Proof of Direct Proof: P → Q is true.

Hypothesis: Assume that P is true i.e. n is odd & m is odd.

Analysis: n = 2k + 1, for some k & m = 2l + 1, for some l.

n + m = (2k + 1) + (2l + 1) = 2(k + l + 1)

Conclusion: R.H.S. = 2(k + l + 1) is divisible by 2.

→ n + m is even integer.

∴ P → Q is true.

Hence, if n is odd and m is odd then n + m is even.

8. Use a direct proof to show that "Every odd integer is the difference of two squares".

Sol: Given: If n is an odd integer then n = s² - t².

Theorem: ∀n [P(n) → Q(n)], where P(n): n is odd integer.

Proof of Direct Proof: P → Q is true.

Hypothesis: Assume that P is true.

Analysis: If n is an odd integer then

n = 2k + 1, where k is some integer.

n = k² + 2k + 1 - k²

n = (k² + 2k + 1) - k²

n = (k + 1)² - k² = s² - t², s = k + 1, t = k.

Conclusion: We observe that n = s² - t².

∴ P → Q is true.

Hence, every odd integer is the difference of two squares.
2. Using direct proof, prove that the sum of two rational numbers is rational.

Sol: Given: If n is rational number & s is rational number, then n+s is rational number.

Theorem: ∀n [P(n) → Q(n)], P(n): n is rational & s is rational.

Q(n): n+s is rational.

Proof: DIRECT PROOF : P⇒Q ⇒ TRUE

Hypothesis: Assume P is TRUE

Analysis: n is rational no: ⇒ n = \( \frac{p}{q} \), q≠0 [Rational no: ⇒

s is rational no: ⇒ s = \( \frac{u}{v} \), v≠0 \( \frac{p}{q}, q≠0 \)

n+s = \( \frac{p}{q} + \frac{u}{v} \) = \( \frac{pv+uq}{qv} \), q≠0, v≠0 ⇒ Q

Conclusion: R.H.S = \( \frac{pv+uq}{qv} \) \( \frac{1}{q} \), v≠0, \( \frac{1}{v} \) \( \frac{q}{v} \) \( \frac{v}{q} \) \( \frac{1}{v} \) = n+s = Rational no:

∴ P⇒Q ⇒ TRUE.

Hence the sum of two rational no: is rational.

5. Give a direct proof that if m' and n' are both perfect squares then nn is also a perfect square.

Sol: theorem: ∀n [P(n) ⇒ Q(n)], P(n): m'n are both Perfect squares.

Q(n): nn are perfect square.

Proof: DIRECT PROOF : P⇒Q ⇒ TRUE

Hypothesis: Assume P is TRUE ⇒ m'n are both Perfect Squares.

Analysis: An integer la' is a perfect square if there is an integer b s.t a = b^2.

m is perfect square ⇒ m = b^2

n is perfect square ⇒ n = e^2

mn = b^2 e^2 = (be)^2

Conclusion: m'n = (be)^2

∴ nn is a perfect square.

∴ P⇒Q ⇒ TRUE

Hence if m'n are both perfect squares then nn is also a perfect square.

HW: 1) Prove that if m+n+p are even integers where m, n, p are integers then m+p is even.

2) Prove that for all integers k & l (if k & l are both odd) then k+l is even & k*l is odd.

TYPE 2: PROBLEMS BASED ON INDIRECT PROOF: CONTRAPOSEIVE.

P⇒Q ⇒ TRUE ⇒ \( \neg P \)⇒\( \neg Q \) ⇒ TRUE.

6. Prove that if n is an integer and 3n+2 is odd then n is odd.

Sol: Given: If n is an integer & 3n+2 is odd then n is odd.

Theorem: ∀n [P(n) ⇒ Q(n)], P(n): n is odd, Q(n): 3n+2 is odd.

∴ P⇒Q ⇒ TRUE.

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Proof: Indirect Proof by Contrapositive: \[ \neg P \Rightarrow \neg T \]

\[ P \Rightarrow T \]

10. \( P = T \) \( \Rightarrow 7P = 3n^2 + 5 \) is even integer.

Hypothesis: \( 70 \) is true \( \Rightarrow 0: n \) is odd integer,

Analysis: \( n \) is even integer \( \Rightarrow n = 2k \) \( k \) is an integer,

\[ 3n^2 + 5 = 3(4k^2 + 6k + 3) = \text{Multiple of 2} \]

\[ \Rightarrow 7P = \text{Even int} \]

Conclusion: \( 3n^2 + 5 = \text{Multiple of 2} \)

\[ \Rightarrow 7P = \text{Even int} \]

Hence \( 70 \Rightarrow 7P = \text{true} \Rightarrow P \Rightarrow Q = \text{true} \)

2. Prove that if \( n \) is an integer and \( n^2 \) is odd then \( n \) is odd.

Solution:

Given: If \( n \) is an integer \( \Rightarrow n^2 \) is odd then \( n \) is odd.

Theorem: \( \forall n \in \mathbb{Z} \) \( \neg P \Rightarrow \neg n \) is odd integer.

Proof: Indirect Proof by Contrapositive

10. \( n \) is even integer \( \Rightarrow n^2 \) is even integer.

Hypothesis: Assume \( 70 \) is true \( \Rightarrow 0: n \) is odd integer,

Analysis: \( n \) is even integer \( \Rightarrow n = 2k \) \( k \) is an integer,

\[ n^2 = (2k)^2 = 4k^2 = 8(2k^2) = 2t \]

Conclusion: \( n \) is an even integer \( \Rightarrow 7P = \text{Even int} \)

\[ \Rightarrow 70 \Rightarrow \neg P \Rightarrow \neg Q = \text{false} \]

If \( n \) is an integer and \( n^2 \) is odd then \( n \) is odd.

3. Show that if \( n \) is an integer \( \Rightarrow n^3 + 5 \) is odd, then \( n \) is even.

Solution:

Theorem: \( \forall n \in \mathbb{Z} \) \( \neg P \Rightarrow \neg n \) is odd integer \( \Rightarrow n^3 + 5 \) is odd.

Proof: Proof by Contrapositive

If \( P \Rightarrow Q \Rightarrow T \)

\[ \neg P \Rightarrow \neg T \]

Hypothesis: Assume \( 70 \) is true \( \Rightarrow 0: n \) is even integer.

Analysis: \( n \) is even \( \Rightarrow n = 2k \), for some integer,

\[ n^3 + 5 = (2k)^3 + 5 = 8k^3 + 12k^2 + 6k + 5 \]

Conclusion: \( n^3 + 5 = \text{Multiple of 2} \)

\[ \Rightarrow 70 \Rightarrow \neg P \Rightarrow \neg Q = \text{false} \]

If \( n \) is an integer \( \Rightarrow n^3 + 5 \) is odd, then \( n \) is odd.

HW

1. Prove that if \( n \) is an integer and \( 5n + 2 \) is odd, then \( n \) is odd.

2. The product of two even integers is an integer.
Show that the statements are equivalent.

$P_1 = n$ is an even integer; $P_2 = (n-1)$ is an odd integer.
$P_3 = n^2$ is an even integer.

So: $P_1 \Rightarrow P_2 \Rightarrow P_3 \Rightarrow \neg P_3 \Rightarrow \neg P_2 \Rightarrow \neg P_1$.

(1) $P_1 \Rightarrow P_2 \Rightarrow T$ [Direct Proof].
   (a) Hypothesis: $P_1$ is true, $P_1 \Rightarrow T$.
   (b) Analysis: $n$ is even integer $\Rightarrow n = 2k$, for some $k$.
   (c) Conclusion: $n-1 = 2k+1$ is odd integer.

(2) $P_2 \Rightarrow P_3 \Rightarrow T$ [Direct Proof].
   (a) Hypothesis: $P_2$ is true, $P_2 \Rightarrow T$.
   (b) Analysis: $n-1 = 2k+1$, for some $k$.
   (c) Conclusion: $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k + 1)$ is even integer.

(3) $P_3 \Rightarrow P_1 \Rightarrow T$ [Indirect Proof].
   (a) Hypothesis: Assume $P_1$ is true, i.e., $P_1 \Rightarrow T$.
   (b) Analysis: $n$ is even integer $\Rightarrow n = 2t$, for some $k$.
   (c) Conclusion: $n^2$ is even integer.

Hence, the statements are equivalent.
Prove that if \( n = ab \) where \( a, b \) are positive integers, then \( n = \sqrt{n} \) or \( b = \sqrt{n} \).

**Solution:**

**Theorem:** \( \forall n \left( \left( \exists m \left[ P(m) \land Q(n) \right] \right) \land \forall (m) \right) \), \( P(m) \Rightarrow n = \sqrt{n} \) or \( b = \sqrt{n} \).

**Proof:** PROOF BY CONTRADICTION: \( \neg P \Rightarrow \neg Q \Rightarrow \neg P \Rightarrow Q \).

**Hypothesis:** Assume \( Q \) is false i.e. \( a = \sqrt{n} \) or \( b = \sqrt{n} \) is false.

**Analysis:** \( a > \sqrt{n} \) or \( b > \sqrt{n} \). Now \( ab > \sqrt{n} \cdot \sqrt{n} = n \).

\( \therefore \ ab > n \Rightarrow [\text{in contradiction to assumption}] \)

**Conclusion:** We observe that \( n \neq ab \) but \( n = ab \).

Thus, our assumption is wrong.

In view of this contradiction, \( P \Rightarrow Q \).

---

Prove that \( \sqrt{2} \) is irrational by giving a proof of contradiction.

**Solution:**

**Theorem:** \( P: \sqrt{2} \) is irrational.

**Proof:** PROOF BY CONTRADICTION.

**Hypothesis:** Assume \( P \) is not true. \( \sqrt{2} \) is rational.

**Analysis:** \( \sqrt{2} = a/b \), there exist integers \( a \) and \( b \) where \( \gcd(a, b) = 1 \), \( a/b \) has no common factors.

\( \Rightarrow a = (\sqrt{2} \cdot b) \Rightarrow b^2 = a^2 \Rightarrow a^2 \) is even \( \Rightarrow a \) is even.

\( \therefore a \) is even, assume \( a = 2c \).

\( b^2 = (\sqrt{2} \cdot b) \Rightarrow b^2 = 4c^2 \Rightarrow b^2 = 2c^2 \Rightarrow b^2 \) is even \( \Rightarrow b \) is even.

**Conclusion:** \( a \) is even \( \land b \) is even \( \Rightarrow ab \) has factor 2. \( \Rightarrow \) which is contradiction to the statement that \( a \) and \( b \) have no common factors.

Thus, our assumption is wrong.

\( \therefore \sqrt{2} \) is irrational.
MA6566 - DISCRETE MATHEMATICS

UNIT-I - LOGICS AND PROOFS

ANNAUNIVERSITY - PART A

1. Find the truth table for the statement $P \Rightarrow q$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$q$</th>
<th>$P \Rightarrow q$</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
<td>F</td>
<td>F</td>
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</tbody>
</table>

2. Construct a truth table for $(P \Rightarrow q) \Rightarrow (q \Rightarrow p)$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$q$</th>
<th>$q \Rightarrow p$</th>
<th>$P \Rightarrow q$</th>
<th>$(P \Rightarrow q) \Rightarrow (q \Rightarrow p)$</th>
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</tbody>
</table>

3. Construct the truth table for $(P \Rightarrow q) \iff (\neg p \Rightarrow q)$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$q$</th>
<th>$P \Rightarrow q$</th>
<th>$\neg p$</th>
<th>$q \Rightarrow (\neg p)$</th>
<th>$(P \Rightarrow q) \iff (\neg p \Rightarrow q)$</th>
</tr>
</thead>
<tbody>
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<td>T</td>
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</table>

4. Using the truth value verify $(P \land q) \land T(P \lor q)$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$q$</th>
<th>$P \land q$</th>
<th>$P \lor q$</th>
<th>$T(P \lor q)$</th>
<th>$(P \land q) \land T(P \lor q)$</th>
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</tr>
</tbody>
</table>

5. Give the truth value of $T \iff (T \land F)$.

<table>
<thead>
<tr>
<th>$T \iff (T \land F)$</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>False</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

6. How many rows are needed for the truth table of the formula $(P \land q) \iff (\neg r \land s) \Rightarrow T$?


7. State the truth value of "If tigers have wings then the earth travels around the sun".

Sol: Propositions: $P$: Tigers have wings  
$q$: Earth travels around the sun.

Symbolic form: $P \Rightarrow q$

Truth Value: Refer Problem 2(b)
write the symbolic representation of "if it rains today, then i buy an umbrella".
sol: propositions: p: it rains today.
 q: i buy an umbrella.
symbolic form: p ⇒ q.

express in symbolic form "the crop will be destroyed if there is a flood."
sol: restatement: if there is a flood then the crop will be destroyed.
proposition: p: there is a flood.
 q: crop will be destroyed.
symbolic form: p ⇒ q.

express in symbolic form "good food is not cheap."

sol: restatement: if there is good food then it is not cheap.
proposition: p: there is good food.
 q: it is cheap.
symbolic form: p ⇒ q.

is the statement "572 iff 5-2 > 0" true?
sol: p: 572, q: 5-2 > 0; p is 't' and q is 't'. . . . . p ⇔ q is true.

is the statement "chennai is in russia iff 3+6=10" true.
sol: p: chennai is in russia (f)
 q: 3+6=10 (f)
 . . . . p ⇔ q is true (t).

let p: i will study discrete mathematics.
 q: i will watch tv.
 r: i am in a good mood.

write the following statement using p, q, r logical connective.

(i) if i donot study discrete and i watch tv, then i am in good mood.
(ii) if i am not in good mood, then i will not watch tv or i will study discrete mathematics.
(iii) if i am in a good mood, then i will study discrete mathematics or i will watch tv.
(iv) i will watch tv and i will not study discrete mathematics if i am in a good mood.

sol: propositions: p: i will study discrete mathematics.
 q: i will watch tv.
 r: i am in a good mood.
symbolic form: (i) (¬p ∨ q) ⇒ r
 (ii) (¬r ⇒ ¬q) ∨ p
 (iii) r ⇒ p
 (iv) (¬q ∨ p) ⇒ r.
10. Write down negation of the following proposition:
(a) The summer in Chennai is hot and sunny.
Sol: The summer in Chennai is not hot or not sunny.
(b) Some people have no two wheelers.
Sol: Every person has a two wheeler.
(c) Every even integer greater than 2 is a sum of two primes.
Sol: There is at least one even integer greater than 2 that is not the sum of two primes.

15. What are the contrapositive, the converse, and the inverse of the conditional statement of the following:
(a) If you work hard then you will be rewarded. [MLB 13]
Sol: Proposition: P: You work hard
q: You will be rewarded.

Given Implication: P \rightarrow q.

Contrapositive: \neg q \Rightarrow \neg p \Rightarrow If you will not be rewarded, then you will not work.

Converse: q \Rightarrow p \Rightarrow If you will be rewarded, then you work hard.

Inverse: \neg p \Rightarrow \neg q \Rightarrow If you will not work hard, then you will not be rewarded.

(b) If it is raining, then I get wet.
Sol: Proposition: P: It is raining.
q: I get wet.

Given Implication: P \rightarrow q.

Contrapositive: \neg q \Rightarrow \neg p \Rightarrow If I do not get wet, then it is not raining.

Converse: q \Rightarrow p \Rightarrow If I get wet, then it is raining.

Inverse: \neg p \Rightarrow \neg q \Rightarrow If it is not raining, then I will not get wet.

16. Find the contrapositive of the inverse of P \rightarrow q; [AML 08]
Sol: Given Implication: P \rightarrow q; Inverse of P \rightarrow q: \neg q \Rightarrow \neg p.
Contrapositive of inverse: \neg p \Rightarrow \neg q.

17. Define Tautology, Contradiction, Contingency with example.
Sol: Tautology: A statement formula which is true always irrespective of the truth values of the individual variables is called Tautology.

Eg: P \lor P \equiv T

Contradiction: A statement formula which is always false.

Eg: P \land \neg P \equiv F

Contingency: A statement formula which is neither Tautology nor Contradiction.

Eg: P \lor \neg P \lor Q is a Contingency.
3. Is $(\neg p \land (p \lor q)) \rightarrow q$ a Tautology?

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$p \lor q$</th>
<th>$(\neg p \lor (p \lor q))$</th>
<th>$\neg p \lor (p \lor q)$</th>
<th>$A \rightarrow q$</th>
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<tbody>
<tr>
<td>T</td>
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</table>

4. Show that $(p \rightarrow (q \rightarrow r)) \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a Tautology.

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<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$q \rightarrow r$</th>
<th>$p \rightarrow (q \rightarrow r)$ (A)</th>
<th>$p \rightarrow q$ (B)</th>
<th>$p \rightarrow r$ (C)</th>
<th>$b \rightarrow c$ (D)</th>
<th>$A \rightarrow D$</th>
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5. Check the following proposition is Tautology. $[(p \rightarrow q) \rightarrow r] \lor \neg p$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$p \rightarrow q$</th>
<th>$(p \rightarrow q) \rightarrow r$ (A)</th>
<th>$\neg p$</th>
<th>$[(p \rightarrow q) \rightarrow r] \lor \neg p$</th>
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<tbody>
<tr>
<td>T</td>
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</table>

All the entries in the resulting column is not T, hence the given proposition is not a Tautology.

6. When do you say that two compound propositions are equivalent?

Sol: $P \leftrightarrow q$ when $P \leftrightarrow q$ is a Tautology.

7. Show that proposition $p \rightarrow q$ and $7p \lor q$ are logically equivalent.

Sol: $T$: $p \rightarrow q \equiv (7p \lor q)$ [Using Truth Table]

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$p \rightarrow q$</th>
<th>$7p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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</tr>
</tbody>
</table>
23. Using Truth Table, show that the proposition \( P \lor (P \land Q) \) is a Tautology.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \lor Q</th>
<th>T(P \lor Q)</th>
<th>\neg T(P \lor Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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24. Show that \((P \rightarrow Q) \land (Q \rightarrow R) \equiv (P \lor Q) \rightarrow R\) logically equivalent.

Sol: L.H.S \equiv \[(P \rightarrow Q) \land (Q \rightarrow R)\]
   \equiv \[\neg P \lor Q \land \neg Q \lor R\]  
   \equiv \[\neg P \lor (\neg Q \lor R)\]  
   \equiv \[\neg (P \land Q) \lor R\]  
   \equiv \neg L.H.S

25. Show that \(P \equiv \neg \neg P\) are equivalent.

Sol: T.P \(P \leftrightarrow \neg \neg P\) [Use Truth Table]

<table>
<thead>
<tr>
<th>P</th>
<th>\neg \neg P</th>
<th>\neg \neg P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
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</tbody>
</table>

\[\text{\therefore \text{Truth value of } P = \text{Truth value of } \neg \neg P\]

\[\therefore P \leftrightarrow \neg \neg P\]

26. Show that \[\neg (P \land (Q \lor R)) \lor (Q \land R) \equiv R\]. Use only the Laws.

Sol: L.H.S \equiv \[\neg (P \land (Q \lor R)) \lor (Q \land R)\]
   \equiv \[\neg (P \land (Q \lor R)) \lor (Q \land R)\]
   \equiv \[\neg (P \land (Q \lor R)) \lor (Q \land R)\]
   \equiv \[\neg (P \land (Q \lor R)) \lor (Q \land R)\]
   \equiv \[\neg (P \land (Q \lor R)) \lor (Q \land R)\]

\[\therefore \text{Given Statement formula is logically Equivalent.}\]

27. Show that \(P \rightarrow (Q \rightarrow R) \equiv (P \land Q) \rightarrow R \equiv P \rightarrow (Q \lor R)\).

Sol: L.H.S : \(1 \equiv P \rightarrow (Q \rightarrow R) \equiv P \rightarrow (Q \lor R) \equiv 2\)  
   \equiv \neg P \lor (Q \lor R)  
   \equiv \neg P \lor (Q \lor R)  
   \equiv \neg P \lor (Q \lor R)  
   \equiv \neg P \lor (Q \lor R)  
   \equiv \neg P \lor (Q \lor R)  
   \equiv \neg P \lor (Q \lor R)  

\[\text{\therefore Condition Law.}\]
Show that \((p \rightarrow q) \equiv (q \rightarrow p) \cdot \neg q\)

\[
\begin{align*}
\text{Sol: L.H.S} & \equiv (p \rightarrow q) \equiv (q \rightarrow p) \cdot \neg q \\
& \equiv (p \neg q) \cdot (q \rightarrow p) \\
& \equiv (p \neg q) \cdot (q \rightarrow p) \\
& \equiv (p \neg q) \cdot (q \rightarrow p) \\
& \equiv (p \neg q) \cdot (q \rightarrow p) \\
& \equiv (p \neg q) \cdot (q \rightarrow p) \\
& \equiv \text{R.H.S.}
\end{align*}
\]

Given statement formulae is logically equivalent.

Show that \((p \lor q) \lor (p \lor q) \equiv \neg q\)

\[
\begin{align*}
\text{Sol: L.H.S} & \equiv (p \lor q) \lor (p \lor q) \\
& \equiv (p \lor q) \lor (p \lor q) \\
& \equiv (p \lor q) \lor (p \lor q) \\
& \equiv (p \lor q) \lor (p \lor q) \\
& \equiv (p \lor q) \lor (p \lor q) \\
& \equiv \text{R.H.S.}
\end{align*}
\]

Given statement formulae is logically equivalent.

Write the equivalent formulae for \(p \land (q \equiv r)\)

\[
\begin{align*}
\text{Sol: L.H.S} & \equiv (p \land (q \equiv r)) \\
& \equiv (p \land (q \equiv r)) \\
& \equiv (p \land (q \equiv r)) \\
& \equiv (p \land (q \equiv r)) \\
& \equiv \text{R.H.S.}
\end{align*}
\]

Given statement formulae is logically equivalent.

Without using Truth Table Show that \((p \rightarrow q) \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow q)]\)

\[
\begin{align*}
\text{Sol: L.H.S} & \equiv (p \rightarrow q) \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow q)] \\
& \equiv \text{R.H.S.}
\end{align*}
\]

Given statement formulae is logically equivalent.
33 Define Tautological Implication. [AIM '08]
Sol: A statement formula A is said to be tautologically imply a statement formula B iff A \rightarrow B is a tautology.

33 Prove the following implication: P \rightarrow (P \lor q).
Sol: 1e \rightarrow T \lor P \rightarrow (P \lor q) is a Tautology.
\[
P \rightarrow (P \lor q) = T \lor q
\]
\[
\equiv (T \lor q) \lor q \quad \text{[Commutative Law]}
\]
\[
\equiv T \lor q \quad \text{[Identity Law]}
\]
\[
\equiv q \lor T \quad \text{[Commutative Law]}
\]
\[
\equiv q
\]
\[
\equiv q \lor q \quad \text{[Identity Law]}
\]
\[
\equiv T
\]
[Condition Law]
[Associative Law]
[Commutative Law]
[Identity Law]

34 Define Dual of a statement formula. [AIM '09]
Sol: Two formulas \( A \) and \( A^* \) are said to be duals of each other if either one can be obtained from other by replacing
\( \land \) by \( \lor \), \( \lor \) by \( \land \), \( T \) by \( F \) and \( F \) by \( T \).

Example (a) Dual \( \left[ (P \land Q) \land R \right] = (P \lor Q) \lor R \)
(b) Dual \( \left[ P \lor (Q \land R) \right] = P \lor (Q \lor R) \)
(c) Dual \( \left[ (P \lor Q) \land (R \lor S) \right] = (P \land Q) \lor (R \land S) \)

35 Define DNF
Sol: Disjunctive Normal Form (DNF) is sum of Elementary Product:
\( (P \land \text{Elementary Product}) \lor (Q \land \text{Elementary Product}) \lor \ldots \lor (S \land \text{Elementary Product}) \)

36 Define CNF
Sol: Conjunctive Normal Form (CNF) is product of Elementary Sum:
\( (P \lor \text{Elementary Sum}) \land (Q \lor \text{Elementary Sum}) \land \ldots \land (S \lor \text{Elementary Sum}) \)

37 Obtain DNF of \( P \land (P \rightarrow Q) \)
Sol: \( P \land (P \rightarrow Q) \equiv P \land (\neg P \lor Q) \quad \text{[Condition Law]}
\]
\[
\equiv (P \land \neg P) \lor (P \land Q) \quad \text{[Distributive Law]}
\]
\[
\equiv \text{Sum of Elementary Product}
\]
\[
\equiv \text{DNF}
\]

38 Obtain Conjunctive Normal Form of \( P \land (P \rightarrow Q) \) [AIM '08]
Sol: \( P \land (P \rightarrow Q) \equiv P \land (\neg P \lor Q) \)
\[
\equiv (P \land \neg P) \lor (P \land Q) \quad \text{[Distributive Law]}
\]
\[
\equiv \text{Product of Elementary Sum}
\]
\[
\equiv \text{CNF}
\]

39 What are the possible truth values for an atomic structure?
Sol: The possible truth values of atomic statements are
\[ \text{TRUE or FALSE} \]
10. Find the conjunctive normal form of \((Q \land (P \lor R)) \lor \neg (C \land V)\).
   Sol: \[Q \land (P \lor R) \lor \neg (C \land V)\]  
   \[\equiv (Q \land (P \lor R)) \lor (\neg C \land \neg V)\]  
   \[\equiv (Q \land P) \lor (Q \land R) \lor (\neg C \land \neg V)\]  
   \[\equiv \text{Product of Elementary Sum}\]  
   \[\equiv \text{CNF}\].

11. Write an equivalent formula for \(P \land (Q \iff R)\) which contains neither the Biconditional nor the Conditional.
   Sol: \(P \land (Q \iff R)\)  
   \[\equiv P \land [(C \implies R) \land (R \implies C)]\]  
   \[\equiv P \land [C \land (C \land R)]\]  
   \[\equiv \text{Equivalent Formula}\].

12. Obtain Principal disjunctive normal forms of \(TP\lor\).
   Sol: \(TP\lor\)  
   \[\equiv (TP \lor \phi) \lor (\phi \lor \phi)\]  
   \[\equiv (TP \lor \phi) \lor (TP \lor \phi)\]  
   \[\equiv (TP \lor (\phi \lor \phi)) \lor (TP \lor (\phi \lor \phi))\]  
   \[\equiv \text{Sum of Minterms}\]  
   \[\equiv \text{PDNF}\].

   Sol: \(\forall x. A(x) \implies A(c)\)

14. When a set of formulae is consistent and inconsistent.
   Sol: Consistent: A set of formulae \(H_1, H_2, \ldots, H_m\) is said to be consistent if their conjunction implies Truth.
   Inconsistent: A set of premises \(H_1, H_2, \ldots, H_m\) is said to be inconsistent if their disjunction implies Contradiction.

15. Define 4-place Predicate.
   Sol: If there are 4 names of objects associated with a predicate, then it is known as 4-place predicate.

16. Show that \(T(PAB)\) follows from \(TPA\lor\).
   Sol: Premises: \(TPA\lor\)
   Conclusion: \(T(PAB)\)
   Method: Indirect Method.
   Additional: \(\neg \text{Conclusion} \equiv \neg T(PAB)\)  
   Premises: \(\neg \text{Conclusion} \equiv \neg T(PAB)\)
### Determine whether C follows logically from the premises

**H1:** \( P \rightarrow Q \) ;  
**H2:** \( P \rightarrow C \) ;  
**C:** \( Q \).  

**Sol:**  
**Premises:** \( P \rightarrow Q \), \( P \)  
**Conclusion:** \( Q \)  

### Is the following argument valid?

If this number is divisible by 4, then it is divisible by 2.  
This number is not divisible by 2. Therefore this number is not divisible by 4.

**Sol:**  
Let \( P: \text{This number is divisible by 4} \).  
\( Q: \text{It is divisible by 2} \).  
**Symbolic form:** \( P \rightarrow Q \), \( \neg Q \).  
**The given argument may be written as:** \( P \rightarrow Q \), \( \neg Q \).  
**By Modus Tollens the argument is valid.** \( \therefore \neg P \)  

### Test the validity of the argument

If it rains today then I will not go for a movie today.  
If I do not go for movie today, then I will go for the movie tomorrow. Therefore, If it rains today, then I will go for the movie tomorrow.

**Sol:**  
**Propositions:** \( P: \text{It is raining today} \).  
\( Q: \text{I will not go for movie today} \).  
\( \therefore \text{I will go for a movie tomorrow} \).  
**Symbolic form:** \( P \rightarrow Q \), \( \neg Q \rightarrow R \).  
**The given argument can be written as:** \( P \rightarrow Q \), \( \neg Q \rightarrow R \).  
**By Hypothetical Syllogism, the argument is valid.** \( \therefore R \rightarrow P \).
If the premises \( \varphi \) and \( \psi \) are inconsistent, then \( \varphi \lor \psi \) is a contradiction from \( \varphi \lor \psi \).

\[ \text{Sol.: Given: } \varphi, \psi \text{ and } \psi \text{ are inconsistent. \therefore } \varphi \lor \psi \equiv \text{False} \]

\[ \text{Proof: } \varphi \lor \psi \Rightarrow \psi \]

Suppose \( \varphi \lor \psi \Rightarrow \psi \) is false.

\[ \therefore \quad \neg \psi \rightarrow \neg \varphi \rightarrow \neg \psi \]

\[ \therefore \varphi \lor \psi \equiv \text{False} \]

\[ \therefore \quad \varphi \lor \psi \Rightarrow \psi \text{ is a contradiction.} \]

Prove that \( p \land q \Rightarrow q \Rightarrow r \Rightarrow r \).

\[ \text{Sol.: Premises: } p \land q, q \Rightarrow r \Rightarrow r \]

Conclusion: \( r \)

<table>
<thead>
<tr>
<th>STEPS</th>
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<td>2</td>
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<tr>
<td>5</td>
<td>( r )</td>
<td>Rule ( P )</td>
</tr>
</tbody>
</table>

Express the statement, "Some people who trust others are rewarded." in symbolic form.

\[ \text{Sol.: Propositions: } P(x) \equiv x \text{ is a person} \]

\[ T(x) \equiv x \text{ trusts others} \]

\[ R(x) \equiv x \text{ is rewarded} \]

Symbolic form: \( \exists x \). \[ P(x) \land T(x) \land R(x) \]

Symbolise the expression "For any \( x \) and any \( y \), if \( x \) is taller than \( y \), then \( y \) is not taller than \( x \)."

\[ \text{Sol.: Let } T \text{ be the predicate 'is taller than'.} \]

\[ T(x, y) \equiv x \text{ is taller than } y \]

\[ \neg T(y, x) \equiv y \text{ is not taller than } x \]

The given statement function: \( \forall x \forall y \). \[ T(x, y) \Rightarrow \neg T(y, x) \]

Write in symbolic form "Every student in this examination hall knows C programming or Java".

| Universe of discourse: Set of all students |

| Propositions: |

\[ A(x) \equiv x \text{ is in this examination hall} \]

\[ B(x) \equiv x \text{ knows C programming} \]

\[ C(x) \equiv x \text{ knows Java} \]

Symbolic form: \( \forall x \). \[ A(x) \lor B(x) \lor C(x) \]

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Express using quantifiers, predicates & logical connectives, the following statement.

(a) Everyone is your friend and is perfect.
(b) Everyone in your class has a cellular phone.
(c) Someone in your class does not play cricket.

Sol: Universe of Discourse: Set of students in your class.

Propositions:
- \( p(x) \): x is your friend.
- \( q(x) \): x is perfect.
- \( r(x) \): x has cellular phone.
- \( h(x) \): x plays Hockey.

Symbolic forms:
- (a) \((\forall x) (p(x) \rightarrow q(x))\)
- (b) \((\forall x) r(x)\)
- (c) \((\exists x) \neg h(x)\)

56 Translate into English, \(\forall x \forall y \left( x > 0 \land y > 0 \right) \Rightarrow \left( xy > 0 \right)\)

Sol: The product of a positive real no. and a negative real no. is a negative real number.

57 Translate the statement \(\forall x \left( c(x) \lor \exists y \left( c(y) \land F(x,y) \right) \right)\)
into English, where c(x): x has a computer, F(x,y): x and y are friends and the universe of discourse is the set of all students in your college.

Sol: Every student in your college has a computer or has a friend who has a computer.

58 What are the negations of the statements \(\forall x \exists y \left( xy = 1 \right)\) so that no negation proceeds a quantifier?

Sol: \(\exists x \forall y \left( xy \neq 1 \right)\)

59 What are the negations of the statements \(\forall x \left( x^2 > 7 \right)\) and \(\exists x \left( x^2 = 2 \right)\)?

Sol:
- \(\neg p(x) = (\forall x) \left( x^2 \leq 7 \right)\)
- \(\neg q(x) = (\exists x) \left( x^2 \neq 2 \right)\)

60 Write the negation of the statement \(\exists x \left( \neg \forall y \left( p(x,y) \right) \right)\)

Sol: \((\forall x) \left( \exists y \left( \neg p(x,y) \right) \right)\)

61 Let \(\alpha(x)\) denote the propositional function 'x is less than 5'. Find the truth value of statement \(\forall x \alpha(x), \exists x \alpha(x)\) if the universe of discourse is \(\{1, 2, 3, 4, 5\}\).

Sol: \(A = \{1, 2, 3, 4, 5\}\) \(\exists x \leq 5 \Rightarrow x \in A\)

We have \(\forall x \alpha(x)\) is false and \(\exists x \alpha(x)\) is true.
4. Given \( P = \{3, 4, 5, 6\} \) state the truth value of the statement 
\( \exists x \in P \) \( x + 3 = 10 \)

5. **Universe of Discourse:** \( P = \{3, 4, 5, 6\} \)

**Statement:** \( P(x) \) \( \iff x + 3 = 10 \)

**Truth value of:** FALSE [there is no \( x \) in \( P \) for which \( x + 3 = 10 \)]

6. Let \( E = \{-1, 0, 12\} \) denote the universe of discourse. Find the truth value of \( \forall x \exists y \ P(x, y) \) [Mark 12]

7. **Universe of Discourse:** \( E = \{-1, 0, 12\} \)

**Statement:** \( P(x, y) \) \( \iff x + y = 1 \)

**Truth value of:** FALSE [there is no \( y \) that works for all \( x \)]

8. Write the truth value of \( \exists x \in P \) \( P(x) \) where \( P(x) \) \( \iff x^2 > 10 \) with \( U = \{1, 2, 3, 4\} \)

9. **Universe of Discourse:** \( U = \{1, 2, 3, 4\} \)

**Statement:** \( P(x) \) \( \iff x^2 > 10 \)

**Truth value of \( \exists x \ P(x) \):** TRUE [\( x = 4 \), \( x^2 > 10 \)]

10. What are the truth values of following quantifications over the set of real numbers?

**a:** \( \forall x \ P(x) \) where \( P(x) \) \( \iff (x+1) - x \)

**b:** \( \exists x \ A(x) \) where \( A(x) \) \( \iff x = x + 1 \)

11. If \( P(x, y) \) be the statement \( x + y = y + x \). What is the truth value of the quantified statement \( \exists x \ P(x, y) \) ?

**Universe of discourse:** Set of all real numbers.

**Statement:** \( P(x, y) \) \( \iff x + y = y + x \)

**Truth value of \( \exists x \ P(x, y) \):** TRUE.

12. Let \( Q(x) \) denote “\( x + y = 0 \)”. What are the truth values of the quantified statements \( \exists y \ Q(x, y) \) \( \forall x \) \( (\exists y) \ Q(x, y) \) \( \forall x \) \( (\exists y) \ Q(x, y) \) ?

**Universe of discourse:** Set of all real numbers.

**Statement:** \( P(x, y) \) \( \iff x + y = 0 \)

**a** Truth value of \( \exists y \) \( (\forall x) \ Q(x, y) \): FALSE

**b** Truth value of \( (\forall x) \ (\exists y) \ Q(x, y) \): TRUE \( \Rightarrow \) [For all real \( x \): there exists \( y \) s.t. \( x + y = 0 \)].

13. Let \( Q(x, y, z) \) be the statement \( x + y = z \). What are the truth values of \( (\forall x) \ (\exists y) \ (\forall z) \ Q(x, y, z) \) ?

**Universe of discourse:** Set of all real \( x \).

**Statement:** \( P(x, y) \) \( \iff x + y = z \)

**a** Truth value of \( (\forall x) \ (\exists y) \ (\forall z) \ Q(x, y, z) \): TRUE

**b** Truth value of \( (\forall x) \ (\exists y) \ (\forall z) \ Q(x, y, z) \): FALSE
What is the truth value of \((\exists x)(\text{P}(x) \Rightarrow \text{Q}(x)) \land T\) where \(\text{P}(x) : x > 2\) and \(\text{Q}(x) : x = 0\). \(T\) is any Tautology. Universe of discourse is \(\mathbb{R}\).

Sol: Universe of Discourse: \(\mathbb{R}\)
Statement: \(\text{P}(x) : x > 2\), \(\text{Q}(x) : x = 0\)
Truth value of \((\exists x)(\text{P}(x) \Rightarrow \text{Q}(x)) \land T\): TRUE

\[\text{E: } x \text{ is allowed to take only value } x = 1. \quad \text{P}(1) : \text{FALSE} \]
\[\text{Q}(1) : \text{FALSE} \]
\[\text{P}(x) \Rightarrow \text{Q}(x) \text{ is TRUE} \]
\[\text{[P}(x) \Rightarrow \text{Q}(x)] \land T \text{ is TRUE}.\]

Over the set of real numbers, what is the truth value of \((\forall x) (\exists y) (xy = 1)\)

Sol: Universe of discourse: Set of real numbers
Statement: \(xy = 1\)
Truth value of \((\forall x) (\exists y) (xy = 1)\): TRUE

\[\text{E: For every real no: } x \text{ there is a real no: } y \text{ s.t } xy = 1\]

Define free and bounded variables.

Sol: 1) BOUNDED VARIABLE: The variable is said to be bound if it is concerned with either universal \((\forall x)\) or existential \((\exists x)\) quantifier.
2) SCOPE: The scope of the quantifier is the formula immediately following the quantifier.
3) The variable which is not concerned with any quantifier is called free variable.

Write the scope of the quantifiers in the formula:
(a) \(\forall x \ (\text{P}(x) \Rightarrow \exists y \ \text{R}(x, y))\)
(b) \(\exists x \ (\text{P}(x) \land \text{Q}(x))\)

Sol: (a) Formula: \(\forall x \ (\text{P}(x) \Rightarrow \exists y \ \text{R}(x, y))\)
Quantifiers: \(\forall x, \exists y\)
Scope of \(\forall x\): \(\text{P}(x) \Rightarrow \exists y \ \text{R}(x, y)\)
\[\exists y : \text{R}(x, y)\]

(b) Formula: \(\exists x \ (\text{P}(x) \land \text{Q}(x))\)
Quantifiers: \(\exists x\)
Scope of \(\exists x\): \(\text{P}(x)\)
x is bound in \(\text{P}(x)\)
x is free in \(\text{Q}(x)\).
Given an indirect proof of the theorem "If $3n+2$ is odd, then $n$ is odd".

So: Given: $3n+2$ is odd.

$\therefore$ $n$ is odd.

Indirect Method:
Assume the conclusion is FALSE and come to a contradiction.
Let $n$ be even

Then $3n+2 = 3(2k) + 2 = 2(3k+1)$. 

$\therefore$ $3n+2$ is even.

$\Rightarrow\Leftarrow$

Our assumption is wrong.

Hence, $n$ is odd and the given implication is true.

Let $n$ be an integer. Prove that if $n^2$ is odd, then $n$ is odd.

So: Here, the conditional to be proved is $p \Rightarrow q$, where

$p$: $n^2$ is odd and $q$: n is odd.

We first prove that the contrapositive $\neg q \Rightarrow \neg p$ is true.

Assume that $\neg q$ is true. Assume that $n$ is not an odd integer.

Then $n=2k$, where $k$ is an integer.

Consequently, $n^2 = (2k)^2 = 4k^2$ = Even integer.

So, $n^2$ is not odd.

$\therefore$ $p$ is false (by $\neg q \Rightarrow \neg p$ is true).

This proves the contrapositive statement $\neg q \Rightarrow \neg p$.

Thus, the proof of the contrapositive $\neg q \Rightarrow \neg p$ serves as an indirect proof of the given statements $p \Rightarrow q$. 

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MA6566-DISCRETE MATHEMATICS

UNIT 1: LOGICS AND PROOFS

ANNAUNIVERSITY-PART B

1. SIMPLIFICATION BY TRUTH TABLE AND WITHOUT TRUTH TABLE

1. Show that \( p \lor (q \land r) \) and \( (p \lor q) \land (p \lor r) \) are logically equivalent.

2. Without using the truth table, prove that \( \sim p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \lor r) \).

3. Prove \( (p \lor q) \land (p \land (\sim q \lor \sim r)) \lor (\sim p \lor q) \lor (\sim p \land r) \) is a Tautology.

4. Prove that \( (p \rightarrow q) \land (q \rightarrow (p \lor r)) \Rightarrow (P \lor r) \rightarrow q \).

5. Prove that \( (p \rightarrow q) \land (q \rightarrow r) \Rightarrow (p \rightarrow r) \).

2. PCNF AND PDNF

1. Without using truth table find the PCNF and PDNF of \( p \rightarrow (q \land p) \land (\sim p \rightarrow (\sim q \land \sim r)) \).

2. Find the Principal Disjunctive normal form of the statement \( (q \lor (p \land r)) \lor (\sim (p \lor r) \land q) \).

3. Obtain the PDNF and PCNF of the statement \( p \lor (\sim p \rightarrow (q \lor (\sim q \rightarrow r))) \).

4. Show that \( (\sim p \rightarrow r) \land (q \leftrightarrow p) \equiv (P \lor q \lor r) \land (P \lor q \lor r) \land (P \lor q \lor r) \land (\sim p \lor q \lor r) \land (\sim p \lor q \lor r) \).

3. THEORY OF INference

1. Show that : \( (P \rightarrow Q) \land (R \rightarrow S), (Q \land M) \land (S \rightarrow N), \sim (M \land N) \) and \( (P \rightarrow R) \Rightarrow \sim P \land (p \rightarrow R) \Rightarrow \sim P \).

2. Show that : \( (p \rightarrow q) \land (r \rightarrow s), (q \rightarrow t) \land (s \rightarrow u), \sim (t \land u) \) and \( (P \rightarrow R) \Rightarrow \sim P \).

3. Prove that the following argument is valid : \( p \rightarrow q, r \rightarrow q, q, r \Rightarrow \sim p \).

4. Prove that the premises \( a \rightarrow (b \rightarrow c), d \rightarrow (b \land c) \) and \( (a \land d) \) are inconsistent.

5. Using indirect method of proof, derive \( p \rightarrow s \) from the premises \( p \rightarrow (q \lor r), q \rightarrow \sim p, s \rightarrow p \) and \( p \).

6. Prove that \( A \rightarrow D \) is a conclusion from the premises \( A \rightarrow B \lor C, B \rightarrow \sim C \) and \( D \rightarrow C \) by using proof.
7. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday," if it is sunny," if we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" leads to the conclusion "we will be home by sunset." [N/D 2012][N/D 2013]

8. Determine the validity of the following argument: If 7 is less than 4, then 7 is not a prime number, 7 is not less than Therefore 7 is a prime number. [M/J 2012]

9. State and explain the proof methods [N/D 2013]

10. Prove that \( \sqrt{2} \) is irrational by giving a proof using contradiction. [N/D 2011][M/J 2013][N/D 2013]

4. QUANTIFIERS

1. Show that \((\forall x)(p(x) \rightarrow q(x)), (\exists y)p(y) \Rightarrow (\exists x)q(x)\). [M/J 2012]

2. Use the indirect method to prove that \((\forall x)(p(x) \lor q(x)) \Rightarrow (\forall x)p(x) \lor (\exists x)q(x)\). [A/M 2011][N/D 2011][A/M 2011]

3. Use the indirect method to prove that the conclusion \(\exists x Q(x)\) follows from the premises \(\forall x[p(x) \rightarrow q(x)]\) and \(\exists y p(y)\). [N/D 2012]

4. Prove that \((\forall x)(p(x) \lor q(x)) \Rightarrow (\forall x)p(x) \lor (\exists x)q(x)\). [M/J 2013]

5. Prove that \(\forall x(p(x) \rightarrow q(x)), \forall x(r(x) \rightarrow \neg q(x)) \Rightarrow \forall x(r(x) \rightarrow \neg p(x))\). [N/D 2010]

6. Show that \((\exists x)(p(x) \land q(x)) \Rightarrow (\exists x)p(x) \land (\exists x)q(x)\). Is the converse true. [N/D 2013]

7. Show that \((\exists x)p(x) \rightarrow \forall x q(x) \Rightarrow (x)(p(x) \rightarrow q(x))\). [M/J 2014]

8. Show that the statement "Every positive integer is the sum of the squares of three integers" is false. [N/D 2011]

9. Verify the validity of the following argument "Every living thing is a plant (or) an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart\). [N/D 2012]

10. Write the symbolic form and negate the following statement:

   (i) Every one who is healthy can do all kinds of work.

   (ii) Some people are not admired by everyone.

   (iii) Every one should help his neighbors, or his neighbors will help him.

   (iv) Everyone agrees with someone and someone agrees with everyone. [A/M 2015]

11. Verify that validating of the following inference, "If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than siva. Therefore, Ram is not more successful than siva. [A/M 2011]
ASSIGNMENT 1
MA6566- DISCRETE MATHEMATICS
UNIT -I

1. \( \neg(p \leftrightarrow q) \leftrightarrow (p \land \neg q) \lor (\neg p \land q) \) using truth table and law.

2. Write the statement in symbolic form
   (a) Some integers are not square of any integers.
   (b) The crop will be destroyed if there is a flood.
   (c) Annu can access the internet from campus only if she is a computer science major or she is not a freshgirl.

3. Give the converse, contrapositive and inverse of the following implication
   (a) If it is raining, then I get wet
   (b) If Raja is a poet, then he is poor

4. Show that \((p \rightarrow q) \land (q \rightarrow r) \rightarrow (p \rightarrow r)\) is a tautology.

5. \((p \land q) \rightarrow r \leftrightarrow (p \rightarrow r) \lor (q \rightarrow r)\)

6. \((Q \rightarrow (P \land \neg P)) \rightarrow (R \rightarrow (P \land \neg P)) \Rightarrow (R \rightarrow Q)\)

7. Obtain the PCNF and PDNF of \((p \rightarrow (q \land r)) \land (\neg p \rightarrow (\neg q \land \neg r))\). Using truth table and law.

8. \(R \land S\) can be derived from \(P \rightarrow Q, Q \rightarrow \neg R, R, P \lor (R \land S)\)

9. Use indirect method and prove that \(p \rightarrow q, r \rightarrow q, s \rightarrow (p \lor r), s \Rightarrow q\).

10. Derive using conditional proof \(P, P \rightarrow (Q \rightarrow R \land S) \Rightarrow Q \rightarrow S\)

11. Show that the following premises are inconsistent:
    A diagnostic message is stored in a buffer or it is retransmitted. A diagnostic message is not stored in the buffers. If a diagnostic message is stored in buffer then it is retransmitted. A diagnostic message is not transmitted.

12. Show that the premises "One student in this class knows how to write programs in JAVA" and "everyone who knows how to write programs in JAVA can get high paying job." Imply the conclusion "someone in class can get a high paying job"