EC6303 - Signals and Systems

Unit-I: Classification of Signals and Systems

Signals:

- A signal is defined as a function of one or more variables which conveys information.
- A signal is a physical quantity that varies with time in general, or any other independent variable.

One-dimensional signal:

- When a function depends on a single independent variable to represent the signal, it is said to be a 1-D signal. (Ex): ECG, EEG, speech signal.

Two-dimensional signal:

- When a function depends on two independent variables to represent the signal, it is said to be a 2-D signal. (Ex): X-ray, x-ray photograph (tomograms).
Multidimensional signal:

When a function depends on more than two independent variables to represent the signal, it is said to be a multidimensional signal.

Example: Speed of wind, air pressure depends on 4 independent variables: latitude, longitude, elevation, and time.

Classification of Signals:

Continuous-time signal:

A signal is \( x(t) \) said to be a continuous-time signal if it is defined at every instant of time \( t \). The amplitude of the signal varies with time. In general, all signals by nature are continuous-time signals. Another common name for a continuous-time signal is an analog signal.

Discrete-time signal:

It is defined at discrete instants of time.
System:

It is a physical device which changes the input signal thereby yielding a new output signal.

$$x(t) \rightarrow \text{System} \rightarrow y(t)$$

Def. It is defined as a physical device that generates a response or output signal for a given input signal. Mathematically, $$y(t) = T[x(t)]$$.

Example problem:

Sketch the signal $$x(t) = e^{-t}$$ for an interval of $$t \leq 2$$. Sample the signal with a sampling period $$T = 0.28$$ and sketch the discrete time signal.

Ans.: $$x(t) = e^{-t}$$, $$x(0) = 1$$, $$x(0.5) = 0.606$$, $$x(1) = 0.36768$$

$$x(1.5) = 0.2237$$, $$x(2) = 0.1353$$.

Sample the signal, $$x(nT) = x(t) \bigg|_{t = nT}$$

$$x(0) = 1$$
$$x(1) = 0.692 = 0.618$$
$$x(2) = e^{-0.4} = 0.67$$
$$x(3) = e^{-0.6} = 0.5088$$
$$x(4) = e^{-0.8} = 0.4099$$
$$x(5) = e^{-1} = 0.3678$$
Elementary CT signals:

To study the system's behavior we use signals as input:

1. Step
2. Impulse
3. Ramp
4. Sinusoidal
5. Exponential functions

Step signal:

Mathematically defined as,

\[
x(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases}
\]

Unit step signal:

If a step function has unity magnitude then it is called a unit step function.

It is defined as,

\[
u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}
\]

Shifted unit step signal:

Advancing, \( u(t+a) = 0 \) for \( t < -a \)

\( u(t+a) = 1 \) for \( t \geq -a \)

Delayed unit step signal:

Delayed by \( a \),

\[
u(t-a) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases}
\]

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Ramp function:

Mathematically defined as,

\[ r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases} \]

Relationship between step and ramp:

The ramp function can be obtained from applying unit step function to an integrator,

\[ r(t) = \int u(t) \, dt = t \] (in the interval \( t = 0 \))

In other words, the unit step function can be obtained by differentiating the unit ramp.

Thus, \( u(t) = \frac{dr(t)}{dt} \).

Impulse function:

It is defined as, \( \int_{-\infty}^{\infty} s(t) \, dt = 1 \).

\( s(t) = 0 \) for \( t \neq 0 \).

It has the amplitude zero for everywhere except at \( t = 0 \). At \( t = 0 \), the amplitude is very high (\( \infty \)).

\[ \text{Area} = \int_{-\infty}^{\infty} s(t) \, dt = 1 \]

Properties of unit impulse:

1. \( \int_{-\infty}^{\infty} x(t) \, s(t) \, dt = x(0) \rightarrow 0 \)

Let us consider the product of \( x(t) \) and \( s(t) \) which is, \( x(t) \cdot s(t) \).
Let the signal $x(t)$ be continuous at $t = 0$, the value of $x(t)$ at $t = 0$, i.e., $x_0$. The impulse exists only at $t = 0$.

Therefore, $x(t) \cdot s(t) = x(t) \cdot s(0)$.

Substitute $s(t)$ in Eq.

$$\int_{-\infty}^{\infty} x(t) s(t) dt = \int_{-\infty}^{\infty} x(t_0) s(t) dt.$$

$$= x(t) \int_{-\infty}^{\infty} s(t) dt.$$

$$= x(t_0).$$

Provided $x(t)$ is continuous at $t = 0$.

$$\int_{-\infty}^{\infty} x(t) s(t) dt = x(t_0).$$

2. Shifting:

$$\int_{-\infty}^{\infty} x(t) s(t-t_0) dt = x(t_0). \rightarrow \square$$

Consider, $x(t) \cdot s(t-t_0) = x(t_0) \cdot s(t-t_0)$. Substitute Eq. $\square$ in Eq.

$$\int_{-\infty}^{\infty} x(t) s(t-t_0) dt = \int_{-\infty}^{\infty} x(t_0) s(t-t_0) dt.$$

$$= x(t) \int_{-\infty}^{\infty} s(t-t_0) dt.$$

$$= x(t_0) \int_{-\infty}^{\infty} s(t) dt.$$

$$= x(t_0).$$
3. Replication:

\[ \int_{-\infty}^{\infty} x(t) \delta(t-t_0) dt = x(t_0) \]

Replace \( t \) by \( t - t_0 \) in Eqn(1), we get,

\[ \int_{-\infty}^{\infty} x(t) \delta(t-t) dt = x(t) \]

Using the even property of impulse function,

\[ \delta(t-t_0) = \delta(t-t) \]

\[ \Rightarrow \int_{-\infty}^{\infty} x(t) \delta(t-t_0) dt = x(t) \]

Convolution of any signal with impulse function gives the original signal.

4. Shifting:

- Delayed by \( t_0 \):
  \[ \delta(t-a) \]
  \[ \Rightarrow x(t-a) \]

- Advanced by \( t_0 \):
  \[ \delta(t+a) \]
  \[ \Rightarrow x(t+a) \]

Unit Ramp function:

The unit ramp function is defined as,

\[ r(t) = t, \text{ for } t \geq 0 \]
\[ = 0, \text{ for } t < 0 \]

Thus, \( r(t) = t \cdot u(t) \)
3. Replication:

\[ \int_{-\infty}^{\infty} x(t) \delta(t-t') \, dt = x(t). \]

Replace \( t \) by \( t' \) in Eq. (3), no get,

\[ \int_{-\infty}^{\infty} x(t') \delta(t-t') \, dt' = x(t') \]

using the even property of impulse function,

\[ \delta(t-t) = \delta(t-2t) \]

\[ \Rightarrow \int_{-\infty}^{\infty} x(t) \delta(t-2t) = x(t) \]

\[ x(t) * \delta(t) = x(t) \] * Convolution

Convolution of any signal with impulse function gives the original signal.

4. Shifting:

* Delayed by \( a \).

\[ \delta(t-a) \]

* Advanced by \( a \).

\[ \delta(t+a) \]

Unit Ramp Function:

The unit ramp function is defined as,

\[ r(t) = t \text{ for } t \geq 0, \]

\[ = 0 \text{ for } t < 0. \]

\[ r(t) = t \text{ for } t \geq 0. \]
3. Replication:

\[ \int_{-\infty}^{\infty} x(t) \delta(t-\omega) \, dt = x(\omega) \]

Replace \( t \) by \( \omega \) in Eqn(3), we get,

\[ \int_{-\infty}^{\infty} x(t) \delta(t-\omega) \, dt = x(\omega) \]

Using the even property of impulse function,

\[ \delta(t-\omega) = \delta(t-\omega) \]

\[ \Rightarrow \int_{-\infty}^{\infty} x(t) \delta(t-\omega) \, dt = x(\omega) \]

Convolution of any signal with impulse function gives the original signal.

4. Shifting:

*Delayed by \( \alpha \):

\[ \delta(t-a) = 0, \quad t = a \]

*Advanced by \( \alpha \):

\[ \delta(t+a) = 0, \quad t = -a \]

Unit Ramp Function:

The unit ramp fn is defined as,

\[ r(t) = t, \quad \text{for } t \geq 0 \]

\[ = 0, \quad \text{for } t < 0 \]

\[ r(t) = t \cdot u(t) \]
**Sinusoidal Signal:**

A continuous-time signal is given by,

\[ x(t) = A \sin(\omega t + \theta) \]

- A: amplitude, \( \omega \): frequency in rad/sec, and
- \( \theta \): phase angle in radians.

**Exponential Signal:**

\[ x(t) = Ae^{at} \]

- \( A \): signal, \( a > 0 \) for growing, \( a < 0 \) for decaying.
- Complex exponential signal:

\[ x(t) = e^{\alpha t} \]

If \( a \) is positive, \( x(t) \) grows exponentially.
If \( a \) is negative, \( x(t) \) decays exponentially.
For \( a = 0 \), the \( x(t) \) is constant.
\[ x(t) = e^{st} = e^{(s+j\omega)t} = e^{st}e^{j\omega t} \]

Using Euler’s identity, we can expand:
\[ e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \]

\[ x(t) = e^{st} (\cos(\omega t) + j\sin(\omega t)) \]

Depending on \( s \) and \( \omega \) values, we get,
(i) If \( s = 0 \), and \( \omega = 0 \), then \( x(t) = 1 \), \( x(t) \) is a DC signal.
(ii) If \( s = 0 \), and \( \omega \), \( x(t) = e^{st} \), which acts as an exponential signal.
(iii) If \( s = 0 \), then \( s = \pm j\omega \) gives \( x(t) = e^{j\omega t} \), \( x(t) \) is a sinusoidal signal with \( \phi = 0 \)
(iv) If \( s = 0 \), with finite \( \omega \), we get exponentially decaying sinusoidal signal.
(v) If \( s > 0 \), with finite \( \omega \), we get exponentially growing sinusoidal signal.

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]

\[ x(t) \]
Representation of DT signals:

1. Graphical rep 2. Functional rep

Graphical:

```
  2 |
  1 |
  0 |
-1 -0.5 0 1 2 3 4 5 n
```

Functional:

```latex
x(n) = \begin{cases} 
  1, & n = -1 \\
  2, & n = 0 \text{ or } 1 \\
  3, & n = 2 \\
  1, & n = 3 \\
  0, & n = 4 \\
\end{cases}
```

Tabular:

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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>2</td>
<td>0.5</td>
<td>1.5</td>
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</table>

Classification of signals:

1. Continuous and deterministic
2. Deterministic & random
3. Periodic & Aperiodic
4. Even & odd
5. Causal & non-causal

Continuous and discrete-time signals:

discussed already in earlier section.

Deterministic: It is a signal exhibiting no uncertainty of value at any given instant.
It can be accurately expressed or predicted by mathematical eqn.

A Signal

Random Signal:

is a signal characterized by uncertainty before its actual occurrence.

Example

Periodic Signal:

A continuous-time signal \( x(t) \) is said to be periodic if it satisfies the condition,

\[
x(t + T) = x(t) \quad \text{for all } t.
\]

The smallest value of \( T \) that satisfies the above condition is known as fundamental period.

Aperiodic Signal:

A signal is aperiodic if it does not satisfy the above condition at least one value of \( t \) and \( n \).

In the case of discrete-time signal, the condition is modified as,

\[
x(n) = x(n + N) \quad \text{for all } n.
\]

Periodic or

N = fundamental period.
The DT signal to be periodic, fundamental frequency.

We must be a rational multiple of the fundamental frequency.

DT signal is periodic.

Symmetric (Even): Anti-symmetric (Odd).

**Even:** \( x(t) \) is even if it satisfies

\[ x(-t) = x(t) \text{ for all } t \]

**Odd:** \( x(t) \) is odd if it satisfies

\[ x(-t) = -x(t) \text{ for all } t \]

Any signal can be represented as the sum of odd and even components.

\[ x(t) = x_e(t) + x_o(t) \]

Replacing \( t \) by \(-t\), gives,

\[ x(-t) = x_e(-t) + x_o(-t) \]

\[ x(-t) = x_o(t) - x_o(t) \rightarrow 0 \]

Adding eqns (a) & (b)

\[ x_e(t) = x_e(t) + x_e(t) \]

\[ x(t) = \frac{1}{2} \left[ x(t) + x(-t) \right] \]
**Energy & Power Signals:**

- **Total Energy of CT Signal $x(t)$ is**
  
  $$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt \text{ Joules}$$

- **Avg. Power of CT Signal $x(t)$ is**
  
  $$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \text{ Watts}$$

  For DT $x[n]$,

  $$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

  $$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

**Definition:**

- A $x[n]$ $x(t)$ is called an energy signal if the energy satisfies $E < \infty$, for an energy signal $P = 0$

- A $x[n]$ $x(t)$ is called a power signal if the power satisfies $E = \infty$ for a power signal $P = \infty$.

The signals that don't satisfy above properties are neither energy nor power signals.
Causal & non-causal:

A signal is causal if,
\[ x(t) = 0 \text{ for } t < 0, \text{ otherwise non-causal.} \]

An anticausal signal is, \[ x(t) = 0 \text{ for } t > 0. \]

In the case of DT signal,
Causal if \[ x(n) = 0 \text{ for } n < 0, \text{ otherwise non-causal.} \]

An anticausal signal is,
\[ x(n) = 0 \text{ for } n > 0 \]

Classification of systems

1. CT & DT, S/M
2. Lumped & distributed parameters
3. Static & Dynamic
4. Causal & Non-causal
5. Linear & Non-linear
6. Time invariant & time variant
7. Stable & Unstable

CT & DT systems:

CTsys: operates on a CT, \( x(t) \) produces a CT output, \( y(t) \).
\[ x(t) \xrightarrow{\text{CTsys}} y(t) \]
\[ y(t) = T[x(t)] \]
Ex: Amplifiers, filters, motors etc.

DTsys: operates on a DT, \( x[n] \), and produces DT output, \( y[n] \).
\[ x[n] \xrightarrow{\text{DTsys}} y[n] \]
Lumped & distributed parameters:

Lumped: in which each & every component is lumped at one point in space. These are described by ordinary differential eqn.

Distributed parameter sys: in which no are fun of space as well as time. These are described by partial differential eqn.

Static & Dynamic sys:

A sys is called static or memory less, if its o/p at any instant depends on the i/p at that instant but not on past or future values of i/p.

\[ y(t) = x^2(t) \]
\[ y(n) = n \times c(n) \]

O/w, sys is dynamic or with memory.

\[ y(t) = \frac{d}{dt} x(t) \]
\[ y(n) = x(n-1) \]

Linear or non-linear:

A sys that satisfies the superposition principle is called linear sys.

Superposition principle states that, response to a weighted sum of i/p can be equal to the weighted sum of i/p corresponding to the each of individual i/p sys. For continuous time linear sys,
where \( \text{ILP } x_1(t) \) produces \( \text{O/LP } y_1(t) = T[x_1(t)] \).

* ILP \( x_2(t) \) produces \( \text{O/LP } y_2(t) = T[x_2(t)] \).

System is linear if:

\[
\text{ILP } a_1 x_1(t) + b x_2(t) \text{ produces } \text{O/LP } a_1 y_1(t) + b y_2(t).
\]

A system doesn't satisfy this principle is called as, non-linear system.

\[
T(a x_1(t) + b x_2(t)) = a T[x_1(t)] + b T[x_2(t)].
\]

From superposition theorem, a zero input results in zero output.

**Causal and Non-causal S/LM:**

Causal S/LM: for which O/LP at any time \( t \) or \( n \) depends on present & past I/LP but not the future I/LP. These are also called as non-anticipative S/LM.

Non-causal S/LM: for which O/LP depends on future values.

**Time Invariant & Time Variant SLM:**

TI S/LM: its I/P-O/P cha does not change with time (relationship).
also be delayed by T units in T - tim.

\[ y(t-T) = T \{ x(t-T) \} \]

If \( y(t) \) due to \( x(t) \) is not equal to \( y(t-T) \); then \( y(t) \) is time variant.

Stable and unstable res.

A res. is said to be bounded if \( x(t) \) is bounded and \( y(t) \) is bounded, stable if \( y(t) \) is only if every bounded \( x(t) \) produces a bounded \( y(t) \).

An \( x(t) \) if \( x(t) \) is bounded if it satisfies the condition, \( |x(t)| \leq Mx < \infty \), for all \( t \).

Similarly for \( y(t) \), \( |y(t)| \leq My < \infty \), for all \( t \).
From the basic knowledge of impulse response of the system we can find whether the system is stable or not.

**Basic operations on signals**

1. Time shifting
2. Time reversal
3. Time scaling
4. Amplitude scaling
5. Multiplication
6. Additive

1. **Time shifting:**
   - Time shifting of \( x(t) \) may delay or advance the signal in time.
   - \( y(t) = x(t - T) \)
   - \( T > 0 \) - advance, \( T < 0 \) - delay

2. **Time reversal:**
   - Time reversal of a signal \( x(t) \) can be obtained by folding the signal about \( t = 0 \).
   - \( x(-t) \) is the reflection of \( x(t) \) about \( t = 0 \).
The conditions are same for DT signals.

3. Time scaling:

\[ y(t) = x(\alpha t) \]

where \( \alpha \) is integer or fraction.

<table>
<thead>
<tr>
<th>( \alpha ) value</th>
<th>compression/Enlargement</th>
</tr>
</thead>
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<tr>
<td>integer</td>
<td>compression</td>
</tr>
<tr>
<td>fraction</td>
<td>enlargement</td>
</tr>
</tbody>
</table>

(i) \( \alpha = 2 \):

\[ y_1(t) = x(\alpha t) \]

\[ y_2(t) = x(\alpha t) \]

(Compression)

(Enlargement)
9. **Amplitude scaling.**

\[ y(t) = A \cdot x(t) \]

- If \( A = 3 \), \( y(t) = 3 \cdot x(t) \)

5. **Signal addition.**

- Sum of 2 CT signals can be obtained by adding their values at every instant.
- Subtraction of 2 CT signals can be obtained by subtracting their values at every instant.

\[ x_1(t) + x_2(t) \]

**Addition**

\[ x_1(t) \]

\[ x_2(t) \]

\[ y(t) = x_1(t) + x_2(t) \]

\[ y_2(t) = x_1(t) - x_2(t) \]
6. **Signal Multiplication:**

Multiplication of two signals can be obtained by multiplying their values at every instant.

\[ x(t) \cdot y(t) \rightarrow y(t) = x(t) \cdot y(t) \]

**Derivation of fundamental period and freq of periodic \( x(t) \):**

For periodic \( x(t) \),
\[ x(t) = x(t+T) \]

Let \( x(t) = A \sin(\omega_0 t + \theta) \rightarrow (1) \)

So, \( x(t+T) = A \sin(\omega_0 (t+T) + \theta) \)

\[ = A \sin(\omega_0 t + \omega_0 T + \theta) \rightarrow (2) \]

\( x(t) \) and \( x(t+T) \) are equal when \( 0 = (2 \pi / \omega_0 ) T + \theta \)

Thus, the fundamental period \( T \) and frequency \( \omega_0 \) of \( x(t) \) can be determined.
$T = \frac{2\pi}{f_0}$  \quad \text{Fundamental period.}

$\omega_c = \frac{2\pi}{T}$  \quad \text{Fundamental freq.}

\( \odot \) \text{ Let } x(t) = e^{j\omega t} \rightarrow (1) \\
\quad x(t+T) = e^{j\omega (t+T)} \\
\quad e^{j\omega T} \cdot e^{j\omega t} \\
\odot \text{ and } e \text{ are equal when, } \\
\quad e^{j\omega T} = 1 \\
\quad \omega T = 2\pi \\
\quad \Rightarrow T = \frac{2\pi}{\omega_0} \\
\quad \Rightarrow \omega_0 = \frac{2\pi}{T} \\
\end{align} \\
\text{Condition for DT S1 to be periodic:} \\
\text{For periodic } x(t), x(t+nT) = x(t+nT) \\
\text{Let } x(t+nT) = A \sin \left( \omega_0 t + \phi \right) \rightarrow (2) \\
\quad x(t+nT) = A \sin \left( \omega_0 t + n\omega_0 T + \phi \right) \rightarrow (3) \\
\odot \text{ and } (2) \text{ are equal when, } \\
\quad \omega_0 n = 2\pi m \\
\quad N = \frac{2\pi m}{\omega_0} \\
\quad \frac{\omega_0}{N} = 2\pi m \\
\end{align} \\
\text{So for DT S1 to be periodic, fundamental freq \( \omega_c \) must be a rational multiple of } \pi. \text{ Otherwise, } \\
\text{DT is aperiodic.} \\
\text{Problem:} \\
\text{CM elementary } \sin \text{ and basic operations:} \\
\begin{align} \\
\text{1. Sketch the following:} \\
\quad \sin \left( \frac{\pi t}{4} \right) \\
\quad \cos \left( \frac{\pi t}{4} \right) \\
\quad \sin \left( \frac{\pi t}{2} \right) \\
\end{align}
2. \[-2u(t-1)
\]

3. \[3y(t-1)
\]

4. \[\pi(t+3)
\]

5. For the signal shown in fig, find the following:
   (i) \(x(t-2)
   
   (ii) \(x(t+3)
   
   (iii) \(x(\frac{t}{2})
   
   (iv) \(x(t+1)
   
   (v) \(x(2t+3)
   
   (vi) \(x(t+3)(t+3)
   
   (vii) \(x(2t+8)
   
   \]

-3 ≤ t ≤ -2, x(t) = 0
-2 ≤ t ≤ -1, x(t) = 2
-1 ≤ t ≤ 0, x(t) = 1.
(iii) $x(t) \rightarrow t$

8) $x\left(\frac{1}{2}t\right)$ can be obtained by compressing $x(t)$ by $\frac{1}{2}$ times.

1.10 Sketch the following

(i) $u(t)-u(t-2)$

(ii) $u(t+\frac{1}{2})$ $u(t)$

(iii) $u(t)$ $u(t-\frac{1}{2})$

(iv) $u(t)$ $u(t)$

1.12 Find the fundamental period $T$ of the following:

(a) $x(t) = e^{j5t}$

It is in the form of $e^{j\omega t}$.

Fundamental period $T = \frac{2\pi}{\omega} = \frac{2\pi}{5} = 0.4\text{ rad/sec}$

(b) $x(t)$ is in the form of $\sin(\omega t)$

Fundamental period $T = \frac{2\pi}{\omega} = \frac{2\pi}{5} = 0.4\text{ rad/sec}$

where, $\omega = \frac{5\pi}{2}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{5\pi}{2}} = \frac{4}{5}$
11. \( 2 \cos (10 \pi t + \pi/6) \)

It is of the form \( A \cos (\omega t + \phi) \)

\[ \omega = 10 \pi \]
\[ T = \frac{2 \pi}{\omega} = 0.2 \text{ sec} \]

1.13 Find whether following NO are periodic or not

(i) \( x(t) = 2 \cos (10t + 1) - \sin (4t - 1) \)

\[ T_1 = \frac{2 \pi}{10} = \frac{\pi}{5} \rightarrow \text{periodic} \]

\[ T_2 = \frac{2 \pi}{4} = \frac{\pi}{2} \rightarrow \text{periodic} \]

\[ \frac{T_1}{T_2} = \frac{\frac{\pi}{5}}{\frac{\pi}{2}} = \frac{2}{5} \rightarrow x(t) \text{ is periodic} \]

\[ T = 5 \text{ sec} \]

(ii) \( x_2(t) = \cos (60t) + \sin (50t) \)

\[ T_1 = \frac{2 \pi}{60} = \frac{\pi}{30} \rightarrow \text{periodic} \]

\[ T_2 = \frac{2 \pi}{50} = \frac{\pi}{25} \rightarrow \text{periodic} \]

\[ \frac{T_1}{T_2} = \frac{\frac{\pi}{30}}{\frac{\pi}{25}} = \frac{5}{6} \rightarrow \text{periodic} \]

\[ T = 5 \text{ sec} \]

(iii) \( 2 \cos t + 3 \sin 2t \)

\[ x(t) \text{ is aperiodic} \]

\[ T_2 = \frac{2 \pi}{2} = \pi \text{ sec} \]

\[ 80, \text{sum is aperiodic} \]
Even & odd signals

1.15 Find even & odd components of following.

(i) \( x(t) = \cos t + \sin t + \cos t + \sin t \)

\( x(-t) = \cos(-t) + \sin(-t) - \cos t - \sin t \)

\( x_{\text{even}}(t) = \frac{1}{2} \left[ x(t) + x(-t) \right] \)

\( x_{\text{even}}(t) = \frac{1}{2} \left[ 2\cos t \right] = \cos t \)

\( x_{\text{odd}}(t) = \frac{1}{2} \left[ x(t) - x(-t) \right] \)

\( = \frac{1}{2} \left[ 2\sin t + 2\cos t \sin t \right] \)

\( = \sin t + \cos t \sin t \).

(ii) \( x(n) = \{-2, 1, 2, -1, 3\} \)

\( x_{\text{even}}(n) = \frac{1}{2} \left[ x(n) + x(-n) \right] \)

\( x_{\text{even}}(0) = \frac{1}{2} \left[ x(0) + x(0) \right] = 0 \)

\( x_{\text{even}}(1) = \frac{1}{2} \left[ x(1) + x(-1) \right] = 0 \)

\( x_{\text{even}}(2) = \frac{1}{2} \left[ x(2) + x(0) \right] = 0.5 \)

\( x_{\text{even}}(3) = \frac{1}{2} \left[ x(3) - x(-3) \right] \)

\( x_{\text{odd}}(0) = \frac{1}{2} \left[ x(0) - x(-0) \right] = 0 \)

\( x_{\text{odd}}(1) = \frac{1}{2} \left[ x(1) - x(-1) \right] = -1 \)

\( x_{\text{odd}}(2) = \frac{1}{2} \left[ x(2) - x(-2) \right] = \frac{3 - (-2)}{2} = 2.5 \)

\( x_{\text{odd}}(3) = \{-2.5, 1, 0, -1, 2.5\} \).
Energy and power signals

1.16 Determine power & R.M.S. Value of \( x(t) = A \cos (\omega_0 t + \phi) \)

\[
P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cos^2 (\omega_0 t + \phi) \, dt
\]

\[
= \lim_{T \to \infty} \frac{A^2}{2} \left[ \int_{-T}^{T} \frac{1 + \cos (2\omega_0 t + 2\phi)}{2} \, dt \right]
\]

\[
= \lim_{T \to \infty} \frac{A^2}{4T} \left[ 2T + \lim_{T \to \infty} \int_{-T}^{T} \cos (2\omega_0 t + 2\phi) \, dt \right]
\]

\[
= \lim_{T \to \infty} \frac{A^2}{4T} (2T) + 0 = \frac{A^2}{2}
\]

**POWER = \( \frac{A^2}{2} \)**

R.M.S. value = \( \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}} \)

1.17 Determine the power and RMS value of followings

(i) \( x_1(t) = 5 \cos (5\omega_0 t + \pi/3) \)

Power = \( 5^2 \times \frac{1}{2} = 12.5 \text{ W} \)

RMS value = \( \sqrt{12.5} = 3.53 \)

(ii) \( x_2(t) = 10 \sin (5\omega_0 t + \pi/4) + 16 \cos (10\omega_0 t + \pi/3) \)

Power = \( \frac{10^2}{2} + \frac{16^2}{2} = 178 \text{ W} \)

RMS = \( \sqrt{178} = 13.34 \)

(iii) \( x_3(t) = 10 \cos 5t + \cos 10t \)
(iv) \( x(t) = e^{j\omega t} \cos 2\omega t \)

\[
P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{j\omega t} \cos 2\omega t|^2 dt
\]

\[
= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (e^{j\omega t} \cos 2\omega t)^2 dt
\]

\[
= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (e^{j\omega t} \cos 2\omega t)(e^{-j\omega t} \cos 2\omega t) dt
\]

\[
= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 2\omega t dt
\]

\[
= \lim_{T \to \infty} \frac{1}{2} \int_{-T}^{T} (1 + \cos 4\omega t) dt
\]

\[
= \lim_{T \to \infty} \left[ \frac{1}{2} t + \frac{1}{2} \sin 4\omega t \right]_{-T}^{T}
\]

\[
= \frac{1}{2} T + \frac{1}{2} \sin 4\omega T - \frac{1}{2} (-T) - \frac{1}{2} \sin 4\omega (-T)
\]

\[
rms = \sqrt{\frac{1}{2}}
\]

10. \( x_5(t) = A e^{j\omega t} \)

Power = \( \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |A e^{j\omega t}|^2 dt \)

\[
= \lim_{T \to \infty} \frac{A^2}{2T} \int_{-T}^{T} dt
\]

\[
= \frac{A^2}{2T} \int_{-T}^{T} dt = \frac{A^2}{2T} \cdot 2T = A^2
\]

\[
rms = A
\]

Energy of signal

1.7 Sketch the following and calculate their energies.

1. \( e^{-\alpha t} u(t) \)

\[
E = \lim_{T \to \infty} \int_{-T}^{T} |e^{-\alpha t} u(t)|^2 dt
\]

\[
= \lim_{T \to \infty} \int_{-T}^{T} e^{-2\alpha t} dt
\]

\[
= \lim_{T \to \infty} \left[ -\frac{e^{-2\alpha t}}{2\alpha} \right]_{-T}^{T}
\]

\[
= \lim_{T \to \infty} \left( -\frac{e^{-2\alpha T}}{2\alpha} + \frac{e^{2\alpha T}}{2\alpha} \right)
\]

\[
= -\frac{e^{-2\alpha T}}{2\alpha} + \frac{e^{2\alpha T}}{2\alpha}
\]

\[
e^{2\alpha T} = \frac{1}{2} \Rightarrow \frac{e^{2\alpha T}}{2\alpha} = \frac{1}{2\alpha}
\]

\[
= \frac{1}{2} - \frac{1}{2\alpha}
\]

\[
e^{-2\alpha T} = \frac{1}{2} \Rightarrow \frac{e^{-2\alpha T}}{2\alpha} = \frac{1}{2\alpha}
\]

\[
= \frac{1}{2} - \frac{1}{2\alpha}
\]

\[
= \frac{1}{2} - \frac{1}{2\alpha}
\]

\[
= \frac{1}{2} - \frac{1}{2\alpha}
\]
1.20 Which of the following are energy signals?

(i) \( u(t) - u(t-1) \)

\[
E = \int_{0}^{1} |x(t)|^2 dt = 1\]

\[
P = \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{2T} x(t)^2 dt = 0.
\]

Since \( E \) finite and \( P = 0 \), \( x(t) \) is energy signal.

(ii) \( x(t) - x(t-2) \)

\[
E = \lim_{T \to \infty} \left[ \frac{1}{2} \int_{0}^{T} x(t)^2 dt + \frac{1}{2} \int_{0}^{T} x(t)^2 dt \right].
\]

\[
P = \lim_{T \to \infty} \left( \frac{t^2}{3} + 4(T-2)^2 \right) = 4.
\]

\[
P = \lim_{T \to \infty} \left( \frac{t^2}{3} + 4(T-2)^2 \right).
\]

\[
E = \lim_{T \to \infty} \left[ \frac{t^2}{3} + 4(T-2)^2 \right] = \infty.
\]

122 Determine energy and power of following signals.

(i) \( x(t) = t \cdot u(t) \)

\[
E = \lim_{T \to \infty} \int_{0}^{T} t^2 dt = \infty.
\]

\[
P = \lim_{T \to \infty} \int_{0}^{T} t^2 dt = \infty.
\]
1.23 Find which of the following are causal or noncausal.

(i) $x(t) = e^{at} u(t)$

$x(t) = 0$ for $t < 0$, so $x(t)$ is causal.

(ii) $x(t) = e^{-at} u(t)$

$x(t) \neq 0$, for $t < 0$, so $x(t)$ is noncausal.

(iii) $x(t) = \text{ sinc } t$

$x(t) = 0$, for $t < 0$, so $x(t)$ is noncausal.

1.24 Consider the signals $x_1(t)$ shown in the figure. Plot $x_1(t-1) + x_1(t+2)$.

Static and dynamic systems

2.1 Find whether the folowing systems are dynamic or not.

(i) $y(t) = x(t-2)$ — Dynamic (depends on past input)

(ii) $y(t) = x(t+2)$ — Dynamic (depends on future input)

(iii) $y(t) = x(t)$ — Static (depends on current input)
Causal / Non-causal Systems

2.2 Check whether the following systems are causal or not.

(i) \( y(n) = \frac{x(n) + 1}{x(n-15)} \) → causal (present output depends on only present part)

(ii) \( y(t) = x(t) + x(t-2) \) → causal

(iii) \( y(t) = x(t) + x(t-2) + x(2-t) \) → non-causal

(iv) \( y(t) = e^{t} \int_{-\infty}^{t} x(\tau) \, d\tau \) → NC.

(v) \( y(t) = x(-t) \) → NC.

Linear / Non-linear Systems

2.3 Check whether the following systems are linear or not.

(i) \( \frac{d}{dt} y(t) + 3y(t) = t^2 x(t) \)

For an input \( x(t) \) corresponding output \( y_1(t) \), then

\[
\frac{d}{dt} y_1(t) + 3y_1(t) = t^2 x_1(t) \rightarrow \Box
\]

\[
\frac{d}{dt} y_2(t) + 3y_2(t) = t^2 x_2(t) \rightarrow \Box
\]

\[
(ax_1(t) + bx_2(t)) \rightarrow a y_1(t) + b y_2(t)
\]

\[
\Rightarrow a \frac{d}{dt} y_1(t) + 3at y_1(t) = a t^2 x_1(t)
\]

\[
\Rightarrow b \frac{d}{dt} y_2(t) + 3bt y_2(t) = b t^2 x_2(t)
\]

\[
\Rightarrow \frac{d}{dt} [ay_1(t) + by_2(t)] + 3t [ay_1(t) + by_2(t)]
\]
\[ \frac{d}{dt} y(t) + 2 \frac{d}{dt} y(t) = x(t) \]
\[ \frac{d}{dt} y_1(t) + 2 y_1(t) = x_1(t) \]
\[ \frac{d}{dt} y_2(t) + 2 y_2(t) = x_2(t) \]
\[ \frac{d}{dt} [a y_1(t) + b y_2(t)] + 2 [a y_1(t) + b y_2(t)] = a x_1(t) + b x_2(t) \]
\[ \text{not a linear fn of weighted sum of inputs} \]
\[ \frac{dy(t)}{dt} + 2 y(t) = x(t) \frac{d}{dt} x(t) \]
\[ \frac{d}{dt} y_1(t) + 2 y_1(t) = x_1(t) \frac{d}{dt} x_1(t) \]
\[ \frac{d}{dt} y_2(t) + 2 y_2(t) = x_2(t) \frac{d}{dt} x_2(t) \]
\[ a \frac{d}{dt} y_1(t) + 2 a y_1(t) = a x_1(t) \frac{d}{dt} x_1(t) \]
\[ b \frac{d}{dt} y_2(t) + 2 b y_2(t) = b x_2(t) \frac{d}{dt} x_2(t) \]
\[ \frac{d}{dt} [a y_1(t) + b y_2(t)] + 2 [a y_1(t) + b y_2(t)] = \frac{d}{dt} x_1(t) + b x_2(t) \]
\[ \text{RHS is not a fn of weighted sum of inputs} \]
\[ \text{no system is noncausal} \]
\[ y(t) = 2 x(t) + \frac{1}{x(t-1)} \]
\[ y_1(t) = 2 x_1(t) + \frac{1}{x_1(t-1)} \]
\[ y_2(t) = 2 x_2(t) + \frac{1}{x_2(t-1)} \]
a y_{1}(n) + b y_{2}(n) = a \left( 2 x_{1}(n) + \frac{1}{a_{1}x_{1}(n)} \right) + b \left( 2 x_{2}(n) + \frac{1}{a_{1}x_{2}(n)} \right)
\hspace{2cm} \frac{1}{a_{1}x_{1}(n)}

y_{3}(n) = \frac{T}{a_{1}x_{1}(n) + b x_{2}(n)} = 2 \left( a_{1}x_{1}(n) + b x_{2}(n) \right) +

\frac{1}{a_{1}(n-1)}

\hspace{2cm} \frac{1}{a_{1}(n-1) + b x_{2}(n-1)}

\hspace{2cm} \frac{1}{a_{1}(n-1) + b x_{2}(n-1)}

\hspace{2cm} \frac{1}{a_{1}(n-1) + b x_{2}(n-1)}

So, Non-causal. $a_{1}(n-1) + b x_{2}(n-1) \neq 0$

2.4: Check whether the given systems are linear or non-linear.

(i) $y(t) = e^{ax(t)}$

\[y_{1}(t) = e^{ax(t)} \hspace{2cm} y_{2}(t) = e^{ax(t)} \rightarrow \Box\]

\[a y_{1}(t) = e^{ax(t)} \hspace{2cm} b y_{2}(t) = e^{bx(t)} \rightarrow \Box\]

\[a y_{1}(t) + b y_{2}(t) = e^{ax(t)} + e^{bx(t)} \rightarrow \Box\]

\[y_{3}(t) = \frac{1}{a_{1}(t) + b_{2}(t)} \rightarrow \Box\]

\[a y_{1}(t) + b y_{2}(t) + y_{3}(t) \rightarrow \Box\]

So, Non-linear

Time invariant / Non-invariant systems.

Steps:

1. $y(t) \rightarrow T(x_{1}(t)) \hspace{2cm} y(t + T) \rightarrow T(x_{1}(t + T))$

2. Delay the input $x_{1}(t)$ by $T$ and denote it by $x_{1}(t - T)$.

3. If $y(t + T) = y(t - T)$ then TI.

4. If $y(t + T) \neq y(t - T)$ then TV.

5. If $y_{c}(n) = T(x_{c}(n)) \hspace{2cm} y_{c}(n) = T(x_{c}(n-k))$

6. If $y_{c}(n-k) = y_{c}(n-k)$ then TI.
For each of the following signals, determine whether the signal is TI or not.

1. \( y(t) = t x(t) \)
   - \( y(t+T) = (t+T) x(t+T) \rightarrow 0 \)
   - \( y(t-T) = (t-T) x(t-T) \rightarrow 0 \)
   ⇒ \( y(t) \) is not TI.

2. \( y(t) = e^{-t} x(t) \)
   - \( y(t+T) = e^{-(t+T)} x(t+T) \rightarrow 0 \)
   - \( y(t-T) = e^{-(t-T)} x(t-T) \rightarrow 0 \)
   ⇒ \( y(t) \) is TI.

Check whether the following are TV or TI.

1. \( \frac{dy(t)}{dt} + 5y(t) = x(t) \)
   - TV.

2. \( y(t) = x(t) + x(t-T) + t x(t-T) \)
   - TV.

3. \( y(t) = x(t+T) + x(t-T) \)
   - TV.

4. \( y(t) = \sin(x(t)) + \cos(x(t)) \)
   - TV.

5. \( y(t) = e^{-t} x(t) \)
   - TV.
Invertibility & Inverse System

\[ \begin{align*}
&x(t) \rightarrow T \rightarrow x(t) = y'(t). \\
&y(t) = T[x(t)] \\
&y'(t) = T^{-1}[y(t)] = T^{-1}(T[x(t)]) = x(t)
\end{align*} \]

To check whether the SLM is stable or not:
1. If the SLM produces zero output for any input then the SLM is not invertible.
2. If the SLM gives same output for different inputs then that SLM is non-invertible.

0. \[ y(t) = Cx(t) \]
   
   For any \( x(t) \) separated by \( C \), SLM gives same output. \( \therefore \) SLM is non-invertible.

1. \[ y(t) = \frac{d}{dt} x(t) \]
   
   If \( x(t) \) is constant, then output is zero. \( \therefore \) Non-invertible.

2. \[ y(t) = x(t + 3) \]
   
   Invertible. Inverse SLM is \( y(t) = x(t + 3) \)

3. \[ y(t) = x^2(t) \]
   
   Non-Invertible.
Stable & unstable

CT Stable \rightarrow \int_{-\infty}^{\infty} h_C(t) dt < \infty

DT Stable \rightarrow \sum_{n=-\infty}^{\infty} |h_C[n]| < \infty.

\( h_C(t) \) is bounded when \( t \rightarrow \infty \) then \( y_C(t) = \infty \)

\( y(t) > y_C(t) \) in (0, \infty).

\( h_C(t) \) is bounded value of \( \sin \) fn. \( u(t - 1) \) to \( t \).

\( \sin \) fn. is multiplied by bounded fns.

\( y(t) \) is Stable.

\( n(t) \rightarrow \text{stable} \)

\( h_C(t) = e^{-3t}H \)

\( \int_{-\infty}^{\infty} |h_C(t)| dt = \int_{-\infty}^{0} e^{-3t} dt + \int_{0}^{\infty} e^{-3t} dt + \int_{-\infty}^{\infty} e^{-3t} dt = \frac{1}{3} (e^{3t})^0 + \left( -\frac{1}{3} \right) (e^{-3t})^\infty \)

\( = \frac{1}{3} (1) - \frac{1}{3} (-1) = \frac{2}{3} < \infty \)

Sim is Stabile.

\( h_C(t) = e^{-3t} u(t+2) \)

\( \int_{-\infty}^{\infty} |h_C(t)| dt = \int_{-2}^{\infty} e^{3t} dt = \frac{1}{3} (e^{3t})^\infty = \infty \)

Unstable.
Tutorial samples of problems on systems

(Problems)

Check whether the following SIs are
(a) static or dynamic (b) linear or non-linear
(c) causal or non-causal (d) time invariant or

time variant.

(i) \( y(t) \frac{d^2 y(t)}{dt^2} + 3t \frac{dy(t)}{dt} + y(t) = x(t) \)

Static or dynamic: The IS is described by
differential eqn. Hence it is a \( \textit{dynamic IS} \)

Linear or non-linear:

Condition: \( T [a x_1(t) + b x_2(t)] = a T[x_1(t)] + b T[x_2(t)] \)

\( x(t) = y(t) \frac{d^2 y(t)}{dt^2} + 3t \frac{dy(t)}{dt} + y(t) \)

\( x_2(t) = y_2(t) \frac{d^2 y_2(t)}{dt^2} + 3t \frac{dy_2(t)}{dt} + y_2(t) \)

\( a x_1(t) = a y_1(t) \frac{d^2 y_1(t)}{dt^2} + 3a t \frac{dy_1(t)}{dt} + a y_1(t) \)

\( b x_2(t) = b y_2(t) \frac{d^2 y_2(t)}{dt^2} + 3 b t \frac{dy_2(t)}{dt} + b y_2(t) \)

\( a x_1(t) + b x_2(t) = a y_1(t) \frac{d^2 y_1(t)}{dt^2} + 3a t \frac{dy_1(t)}{dt} + a y_1(t) \)

\( + b y_2(t) \frac{d^2 y_2(t)}{dt^2} + 3 b t \frac{dy_2(t)}{dt} + b y_2(t) \)

\( a y_1(t) \frac{d^2 y_1(t)}{dt^2} + b y_2(t) \frac{d^2 y_2(t)}{dt^2} + 3t \left[ a \frac{dy_1(t)}{dt} + b \frac{dy_2(t)}{dt} \right] \)
boxed position is not a to of weight sum of
clp. Hence superposition principle is not
satisfied, and the s/n is non-linear.
causal or non-causal?
The clp depends on the present i/p only.
Hence the s/n is causal.
Time-invariant or Time Variant?
The coefficients of the differential egn are
function of time. Hence the s/n is time-variant.

\[ \frac{d^3y(t)}{dt^3} + 4 \frac{dy(t)}{dt^2} + 5dy(t) + ay^3(t) = x(t) \]

static or dynamic: The s/n is described by a
differential egn. Hence it is a dynamic s/n.
Linear or nonlinear: The condition for superposition
theorem,
\[ T(ax_1(t) + bx_2(t)) = aT(x_1(t)) + bT(x_2(t)) \]

\[ ax_1(t) = \frac{d^3y(t)}{dt^3} + 4 \frac{dy(t)}{dt^2} + 5dy(t) + ay^3(t) \]

\[ bx_2(t) = \frac{d^3y(t)}{dt^3} + 4 \frac{dy(t)}{dt^2} + 5dy(t) + ay^3(t) \]

\[ \frac{d^3y(t)}{dt^3} + 4 \frac{dy(t)}{dt^2} + 5dy(t) + ay^3(t) \]

\[ ax_1(t) + bx_2(t) \]
\[
\frac{d^3}{dt^3} [a y_1(t) + b y_2(t)] + 4 \frac{d^2}{dt^2} [a y_1(t) + b y_2(t)] \\
+ 5 \frac{d}{dt} [a y_1(t) + b y_2(t)] + 2 a y_1(t) + 2 b y_2(t)
\]

Shaded portion in the RHS is to be weighted sum of square of inputs. Superposition principle is not satisfied. Thus the system is **non-linear**

Causal or non-causal? The output depends on present input only. Therefore the system is **causal**

Time variant or Time variant? The coefficients of differential equations are constant. Hence the system is **time-invariant**

\[y[n] = x[n] + x[n-1]\]

Static or dynamic? The output depends on the past values of input. So, the system is **dynamic**

Linear or non-linear?

\[y[n] = a x[n] + b x[n-1]\]

\[y[n] = a x[n] + b x[n-1]\]

\[b y_1[n] = b x[n] + b x[n-1]\]

\[a x[n] + b x[n-1] = a x[n] + b x[n-1]\]

Hence the system is **linear**
Causal or non-causal?

The output depends on present and past values of input. Hence the system is **causal**.

Time-invariant or variant?

The output due to delayed input is

\[ y[n-k] = T[x[n-k]] = x[n-k] \cdot \delta[n-1-k] \]

The delayed output is

\[ y[n-k] = x[n-k] \cdot \delta[n-1-k] \]

\[ y[n-k] = y[n-k] \]

The system is **time-invariant**.

The system is **dynamic, non-linear, causal and time-invariant**.

(iv) \[ y[n] = \cos[\alpha x[n]] \]

Static or dynamic?

The output at any instant depends on the input at that instant. Hence the system is **static**.

Linear or non-linear?

\[ y_1[n] = \cos(x_1[n]) \]

\[ y_2[n] = \cos(x_2[n]) \]

\[ a_1 y_1[n] + b y_2[n] = a \cos(x_1[n]) + b \cos(x_2[n]) \]

\[ a \cdot \text{LHS} \neq \text{RHS} \]

Hence the system is **non-linear**.
The op depends on present input only. Hence the system is **causal**.

**Time-invariant or time-variant?**

\[ y[n, k] = T[x(n - k)] = \cos(\omega (n - k)) \]

The delayed op is,

\[ y[n - k] = \cos(\omega n - k \omega) \]

The system is **time-invariant**.

The system is **static, non-linear and time-invariant**.

\[ y[n] = \text{sgn}(\omega n) \]

**Solution:**

\[
\text{sgn}(\omega n) = \\
\begin{cases} 
1 & \text{for } n > 0 \\
-1 & \text{for } n < 0 
\end{cases} 
\]

Static or **dynamic**?

The op always depends on present input only. Hence the system is **static**.

Linear or **non-linear**?

\[ y[n] = T[x(n)] = \text{sgn}(\omega n) \]

For \(x(n)\) \(\Rightarrow\) \(y[n] = T[x(n)] = \text{sgn}(\omega n)\)
\[ T(z_{1+n} + b \cdot z_n) = \text{sgn}(z_{1+n}) + b \cdot \text{sgn}(z_n) \]

The weighted sum of \( z \)

\[ T(z_{1+n} + b \cdot z_n) = \text{sgn}(z_{1+n}) + b \cdot \text{sgn}(z_n) \]

\[ a \cdot y_{1+n} + b \cdot y_n = \text{sgn}(a \cdot z_{1+n} + b \cdot z_n) \]

LHS \neq \text{RHS}

Therefore the \( SLM \) is \text{nonlinear}.

\[ \text{causal or noncausal} \]

The \( \text{OLP} \) \( y(n) \) depends on present \( iLP \).

hence the \( SLM \) is \text{causal}.

\[ \text{Time invariant or time-variant} \]

\[ 0 \cdot n, \quad y(n) = \text{sgn}(z_{1+n}) \]

The \( OLP \) due to delayed \( iLP \).

\[ y(n+k) = T(z_{1+n-k}) = \text{sgn}(z_{1+n-k}) \]

The delayed \( OLP \)

\[ y(n+k) = \text{sgn}(z_{n+k}) \]

\[ y(n+k) = y(n+k) \]

Hence the \( SLM \) is \text{time-invariant}.

Hence the \( SLM \) is \text{static, non-linear, causal and time-invariant}.