DIGITAL SIGNAL PROCESSING
UNIT 1 – SIGNALS AND SYSTEMS

1. What is a continuous and discrete time signal?

   Continuous time signal: A signal x(t) is said to be continuous if it is defined for all time t. Continuous time signal arise naturally when a physical waveform such as acoustic wave or light wave is converted into a electrical signal.

   Discrete time signal: A discrete time signal is defined only at discrete instants of time. The independent variable has discrete values only, which are uniformly spaced. A discrete time signal is often derived from the continuous time signal by sampling it at a uniform rate.

2. Give the classification of signals?

   - Continuous-time and discrete time signals
   - Even and odd signals
   - Periodic signals and non-periodic signals
   - Deterministic signal and Random signal
   - Energy and Power signal

3. What are the types of systems?

   Continuous time and discrete time systems
   - Linear and Non-linear systems
   - Causal and Non-causal systems
   - Static and Dynamic systems
   - Time varying and time in-varying systems
   - Distributive parameters and Lumped parameters systems
   - Stable and Un-stable systems.

4. What are even and odd signals?

   Even signal: continuous time signal x(t) is said to be even if it satisfies the condition x(t)=x(-t) for all values of t.

   Odd signal: he signal x(t) is said to be odd if it satisfies the condition x(-t)=-x(t) for all t.

   In other words even signal is symmetric about the time origin or the vertical axis, but odd signals are anti-symmetric about the vertical axis.

5. What are deterministic and random signals?

   Deterministic Signal: deterministic signal is a signal about which there is no certainty with respect to its value at any time. Accordingly, we find that deterministic signals may be modeled as completely specified functions of time.

   Random signal: random signal is a signal about which there is uncertainty before its actual occurrence. (e.g.) The noise developed in a television or radio amplifier is an example for random signal.

6. What are energy and power signal?

   Energy signal: Signal is referred as an energy signal, if and only if the total energy of the signal satisfies the condition 0<E<∞.

   Power signal: Signal is said to be power signal if it satisfies the condition 0<P<∞.
7. What are elementary signals and name them?
The elementary signals serve as a building block for the construction of more complex signals. They are also important in their own right, in that they may be used to model many physical signals that occur in nature.

There are five elementary signals. They are as follows
- Unit step function
- Unit impulse function
- Ramp function
- Exponential function
- Sinusoidal function

8. What are time invariant systems?
A system is said to be time invariant system if a time delay or advance of the input signal leads to an identical shift in the output signal. This implies that a time invariant system responds identically no matter when the input signal is applied. It also satisfies the condition
\[ R\{x(n-k)\} = y(n-k). \]

9. What do you mean by periodic and non-periodic signals?
A signal is said to be periodic if \( x(n+N) = x(n) \), Where N is the time period.
A signal is said to be non-periodic if \( x(n+N) = -x(n) \).

10. Determine the convolution sum of two sequences \( x(n) = \{3, 2, 1, 2\} \) and \( h(n) = \{1, 2, 1, 2\} \)

11. Define time variant and time invariant system.
A system is called time invariant if its output, input characteristics does not change with time. A system is called time variant if its input, output characteristics changes with time.

12. Define linear and non-linear system.
Linear system is one which satisfies superposition principle. Superposition principle:
The response of a system to a weighted sum of signals be equal to the corresponding weighted sum of responses of system to each of individual input signal.
\[ i.e., T[ax_1(n)+ax_2(n)] = ax_1(n) + ax_2(n) \]
A system, which does not satisfy superposition principle, is known as non-linear system.

13. Define causal and non-causal system.
The system is said to be causal if the output of the system at any time ‘n’ depends only on present and past inputs but does not depend on the future inputs.
\[ e.g.: y(n) = x(n) - x(n-1) \]
A system is said to be non-causal if a system does not satisfy the above definition.

14. What are the steps involved in calculating convolution sum?
The steps involved in calculating sum are
- Folding
- Shifting
- Multiplication
- Summation

15. Define causal LTI system.
The LTI system is said to be causal if \( h(n) = 0 \) for \( n < 0 \)

**16. Define stable LTI system.**

The LTI system is said to be stable if its impulse response is absolutely summable.

**17. What are the properties of convolution sum?**

The properties of convolution sum are

- **Commutative property**
  The commutative law can be expressed as \( x(n) * h(n) = h(n) * x(n) \)

- **Associative law**
  The associative law can be expressed as \( [x(n) * h1(n)] * h2(n) = x(n) [h1(n) * h2(n)] \)
  Where \( x(n) \) – input, \( h1(n), h2(n) \) - impulse response

- **Distributive law**
  The distributive law can be expressed as
  \( x(n) * [h1(n) + h2(n)] = x(n) * h1(n) + x(n) * h2(n) \)

**18. Define Z-transform**

Z-Transform can be defined as
\[
X(Z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}
\]

**19. Define Region of convergence**

The region of convergence (ROC) of \( X(Z) \) is the set of all values of \( Z \) for which \( X(Z) \) attain final value.

**20. State properties of ROC**

- The ROC does not contain any poles.
- When \( x(n) \) is of finite duration then ROC is entire \( Z \)-plane except \( Z = 0 \) or \( Z = \infty \)
- If \( X(Z) \) is causal, then ROC includes \( Z = \infty \)
- If \( X(Z) \) is non-causal, then ROC includes \( Z = 0 \)

**21. Continuous time and Discrete time signals.**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Continuous Time (CTS)</th>
<th>Discrete Time (DTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>This signal can be defined at any time instance &amp; they can take all values in the continuous interval((a, b)) where (a) can be (-\infty) &amp; (b) can be (\infty)</td>
<td>This signal can be defined only at certain specific values of time. These time instance need not be equidistant but in practice they are usually takes at equally spaced intervals.</td>
</tr>
<tr>
<td>2</td>
<td>These are described by differential equations.</td>
<td>These are described by difference equation.</td>
</tr>
<tr>
<td>3</td>
<td>This signal is denoted by (x(t)).</td>
<td>These signals are denoted by (x(n)) or</td>
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**22. Analog and digital signal**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Analog signal</th>
<th>Digital signal</th>
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<tbody>
<tr>
<td>1</td>
<td>These are basically continuous time &amp; continuous amplitude signals.</td>
<td>These are basically discrete time signals &amp; discrete amplitude signals. These signals are basically obtained by sampling &amp; quantization process.</td>
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</tbody>
</table>
ECG signals, Speech signal, Television signal etc. All the signals generated from various sources in nature are analog.

All signal representation in computers and digital signal processors are digital.

PART – B (16 Marks)

1. Explain in detail about the classification of discrete time systems. (16)

2. (a) Describe the different types of discrete time signal representation. (6)
   (b) Define energy and power signals. Determine whether a discrete time unit step signal
   \[ x(n) = u(n) \]
   is an energy signal or a power signal. (10)

3. (a) Give the various representation of the given discrete time signal
   \[ x(n) = \{-1,2,1,-2,3\} \]
   in Graphical, Tabular, Sequence, Functional and Shifted functional. (10)
   (b) Give the classification of signals and explain it. (6)

4. (a) Draw and explain the following sequences:
   i) Unit sample sequence
   ii) Unit step sequence
   iii) Unit ramp sequence
   iv) Sinusoidal sequence
   v) Real exponential sequence (10)
   (b) Determine if the system described by the following equations are causal or noncausal
   i) \[ y(n) = x(n) + (1 / (x(n-1))) \]
   ii) \[ y(n) = x(n^2) \] (6)

5. Determine the values of power and energy of the following signals. Find whether the signals
   are power, energy or neither energy nor power signals.
   i) \[ x(n) = \frac{1}{3}n u(n) \]
   ii) \[ x(n) = e^{j(\pi/2)n + (\pi/4)} \]
   iii) \[ x(n) = \sin (\pi/4)n \]
   iv) \[ x(n) = e^{2n} u(n) \] (16)

6. (a) Determine if the following systems are time-invariant or time-variant
   i) \[ y(n) = x(n) + x(n-1) \]
   ii) \[ y(n) = x(-n) \] (4)
   (b) Determine if the system described by the following input-output equations are linear or non-linear.
   i) \[ y(n) = x(n) + (1 / (x(n-1))) \]
   ii) \[ y(n) = x^2(n) \]
   iii) \[ y(n) = nx(n) \] (12)

7. Test if the following systems are stable or not.
   i) \[ y(n) = \cos x(n) \]
   ii) \[ y(n) = ax(n) \]
   iii) \[ y(n) = x(n) en \]
   iv) \[ y(n) = ax(n) \] (16)

8. (a) Explain the principle of operation of analog to digital conversion with a neat diagram. (8)
   (b) Explain the significance of Nyquist rate and aliasing during the sampling of continuous
time signals. (8)
9. (a) List the merits and demerits of Digital signal processing. (8)
(b) Write short notes about the applications of DSP. (8)

10. (a) Find the convolution of the following sequences
   i) \( x(n) = u(n) \quad h(n) = u(n-3) \) (8)
   ii) \( x(n) = \{1,2,-1,1\} \quad h(n) = \{1,0,1,1\} \) (8)

   (b) Determine the response of the causal system \( y(n) - y(n-1) = x(n) + x(n-1) \) to inputs \( x(n) = u(n) \) and \( x(n) = 2^{-n} u(n) \). (8)

11. (a) Determine the solution of the difference equation
    \[ y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \quad \text{for} \quad x(n) = 2n \quad u(n) \] (8)

    (b) Determine the response \( y(n) \), \( n \geq 0 \) of the system described by the second order difference equation
    \[ y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1) \] when the input is \( x(n) = (-1)^n u(n) \) and the initial condition are \( y(-1) = y(-2) = 1 \). (8)

12. State and prove any two properties of z-transform. (8)

13. Find the z-transform and ROC of the causal sequence \( X(n) = \{1,0,3,-1,2\} \) (8)

14. Find the z-transform and ROC of the anticausal sequence \( X(n) = \{-3,-2,-1,0,1\} \) (8)

15. Determine the z-transform and ROC of the signal
   (a) \( x(n) = anu(n) \) and
   (b) \( x(n) = -bnu(-n) \) (16)

16. Determine the response \( y(n) \) of the system described by the second order difference equation
    \[ y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1) \] when the input is \( x(n) = (-1)^n u(n) \) and the initial condition are \( y(-1) = y(-2) = 1 \). (8)

17. State and prove the following properties of z-transform.
   i) Time shifting
   ii) Time reversal
   iii) Differentiation
   iv) Scaling in z-domain

18. Determine the inverse z-transform of \( x(z) = \frac{1+3z}{1+3z-1+2z-2} \) for \( z > 2 \) (8)

19. Find the inverse z-transform of \( x(z) = \frac{z^2+z}{z-1}(z-3) \), ROC: \( z > 3 \). Using (i) Partial fraction method, (ii) Residue method and (iii) Convolution method (16)

20. Determine the unit step response of the system whose difference equation is
    \[ y(n) - 0.7y(n-1) + 0.12y(n-2) = x(n-1) + x(n-2) \quad \text{if} \quad y(-1) = y(-2) = 1. \] (8)

21. Determine the convolution sum of two sequences \( x(n) = \{3,2,1,2\} \), \( h(n) = \{1,2,1,2\} \) (8)

22. Find the convolution of the signals \( x(n) = 1 \quad n = -2,0,1 \quad 2 \quad n = -1 = 0 \) elsewhere \( h(n) = \delta(n)-\delta(n-1)+\delta(n-2)-\delta(n-3) \) (8)