When an external force acts on a body, the body tends to undergo some deformations.

* Due to the cohesion b/w the molecules, the body resists deformation.

* This resistance by which material of the body opposes the deformation is known as "Strength of material".

* **Unit - I**

Stress, strain & Deformation of Solids:

**Stress:**

* The force of resistance per unit area offered by a body against deformation is known as stress.

* The external force acting on the body is called the load or force.
In simple words, it’s defined as the internal resistance which the body offers to meet with the load is called stress.

Mathematically, stress is written as

$$\sigma = \frac{P}{A}$$

Where $\sigma$ = Stress

$P$ = External force or load and $A$ = Cross-sectional area.

Unit of stress = $N/m^2$ (or) $N/mm^2$.

Strain ($\varepsilon$):

When a body is subjected to some external force, there is some change of dimension of the body. The ratio of the change of dimension of the body to the original dimension is known as strain.

Strain is dimensionless.

Types of Strains:

1. Tensile
2. Compressive & Shear
Types of Stresses:

- Normal stress \( \sigma_N \) is the stress which acts in a direction perpendicular to the area.
  - It is represented by \( \sigma_N \).
  - The normal stress is further divided into:
    1. Tensile stress.
    2. Compressive stress and

Tensile Stress:

- The stress induced in a body when subjected to two equal and opposite pulls.

\[
P \leq \frac{1}{2} \sigma \leq P
\]

- As a result of which there is an increase in length, is known as Tensile Stress.

\[
\text{Tensile stress} = \sigma = \frac{\text{Tensile load } (P)}{A}
\]

\[
\therefore \sigma = \frac{P}{A}
\]
Tensile Strain:

The ratio of increase in length to the original length is known as tensile strain.

It is denoted by \( \varepsilon \).

\[
\varepsilon = \frac{\text{Increase in length}}{\text{Original length}} = \frac{SL}{L}
\]

Compressive Stress:

The stress induced in a body when subjected to two equal and opposite pushes.

\[
\sigma = \frac{\text{Resisting force (R)}}{\text{Area (A)}} = \frac{P}{A}
\]

Compressive Strain:

\[
\varepsilon = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{SL}{L}
\]
Shear Stress:

- The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section.

- As a result of which the body tends to shear off across the section is known as Shear Stress.

\[ \tau = \frac{P}{A} \]

It is denoted by the symbol ‘\( \tau \)’.

Shear stress, \( \tau = \frac{\text{Shear Resistance}}{\text{Shear area}} = \frac{P}{A} = \frac{P}{\ell} \)

Shear Strain (\( \phi \)):

\[ \phi = \frac{\text{Transversal displacement}}{\text{Distance}} \]

\[ \phi = \frac{S_{1}}{h} \]
Elasticity:

- When an external force acts on a body, the body tends to undergo some deformation.

- If the external force is removed & the body comes back to its original shape & size, which means the deformation disappears completely, the body is known as elastic.

- This property, by virtue of which certain materials return back to their original position after the removal of the external force is called elasticity.

Elastic limit:

- The body will regain its previous shape & size only when the deformation caused by the external force is within a certain limit.

- Thus, there is a limiting value of force up to & within which, the deformation completely disappears on the removal of the force.

- The value of stress corresponding to the limiting force is known as the elastic limit of the material.
Hooke's Law & Elastic Modulus:

Hooke's Law states that when a material is loaded within elastic limit, the stress is proportional to the strain is a constant within elastic limit.

This constant is known as modulus of elasticity or modulus of rigidity or elastic moduli.

Young's Modulus (or) Modulus of elasticity:

The ratio of tensile stress or compressive stress to the corresponding strain is a constant.

This ratio is known as Young's modulus or Modulus of Elasticity & it is denoted by $E$.

$$E = \frac{\text{Tensile Stress (or)}}{\text{Tensile Strain}} = \frac{\text{Compressive Stress}}{\text{Compressive Strain}}.$$

$E = \frac{\sigma}{\varepsilon}.$

Shear Modulus (or) Modulus of Rigidity:

The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as modulus of rigidity or modulus of shear. It's denoted by $G$ or $G_{\text{mod}}.$

$$G = \frac{\tau}{\gamma}.$$
Factor of Safety:

It is defined as the ratio of ultimate tensile stress to the working (or permissible) stress. Mathematically, it's written as

\[ \text{factor of safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress}} \]

1. Longitudinal Strain:

When a body is subjected to an axial load, there is an increase in the length of the body. But at the same time, there is a decrease in other dimensions of the body at right angles to the line of action of the applied load.

\[ \text{Longitudinal strain} = \frac{\delta_l}{l} \]

\[ \delta_l = \text{Increase in length} \]

\[ l = \text{Length of the body} \]

Lateral Strain:

The strain at right angles to the direction of the applied load is known as lateral strain.
The length of the bar will increase while the breadth & depth will decrease.

\[ \text{lateral strain} = \frac{\Delta b}{b} \text{ or } \frac{\Delta d}{d} \]

\( \Delta b \) = decrease in breadth, \( \Delta d \) = decrease in depth.

Poisson's ratio:

The ratio of the lateral strain to the longitudinal strain is a constant for a given material. When the material is stressed within the elastic limit, it is generally denoted as \( \mu \).

\[ \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}} \]

Relationship between stress & strain (2.0D)

\[ e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \]

\[ e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} \]

where, \( e_1 \) = total strain in x direction, \( e_2 \) = total strain in y direction, \( \sigma_1 \) = normal stress in x direction, \( \sigma_2 \) = normal stress in y direction.
for (8.1) relationship 

\[ e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_2}{E} \]

\[ e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} \]

\[ e_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \]

Pb : - 0

A tensile test was conducted on a mild steel.

The following data was obtained from the test:

- Dia. of steel bar: 3 cm
- Gauge length of bar: 20 cm
- Load at elastic limit: 250 kN
- Extension at a load of 150 kN: 0.21 mm
- Max. load: 380 kN
- Total expansion: 60 mm
- Diameter of the rod at the failure: 2.25 cm

Soln:-

Area of the rod: 

\[ A = \frac{\pi}{4} D^2 \]

\[ = \frac{\pi}{4} \left( 3 \times 10^{-2} \right)^2 \]

\[ = 7.0685 \times 10^{-4} \text{ m}^2 \]
Stress = \frac{\text{Load}}{\text{Area}} = \frac{150 \times 1000}{7.0685 \times 10^{-4}} \Rightarrow 81220.9 \times 10^4 \text{N/m}^2$

Strain = \frac{\text{Increase in length}}{\text{Original length}} = \frac{0.21 \times 10^{-3}}{20 \times 10^{-2}} = 0.00105$

Young's Modulus, E = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \frac{81220.9 \times 10^4}{0.00105} = 2.0209523 \times 10^7 \text{N/m}^2 \Rightarrow 202.09526 \text{GPa}$

b) Stress at elastic limit

\text{Stress} = \frac{2.50 \times 10^3}{7.0685 \times 10^{-4}} = 3.5368 \times 10^4 \text{N/m}^2$

\leq 3.5368 \text{GPa}$

c) % elongation:

% elongation = \frac{\text{Total increase}}{\text{Original length}} \times 100$

= \frac{60 \times 10^{-3}}{20 \times 10^{-3}} \times 100 = 30.1$

d) Percentage decrease in area

% decrease = \left(\frac{\text{Original Area} - \text{Area at failure}}{\text{Original Area}}\right) \times 100$

= \left(\frac{(3 \times 10^{-2})^2 - \left\{\frac{2.25 \times 10^{-2}}{2}\right\}^2}{(3 \times 10^{-2})^2}\right) \times 100$

= 5.25%
Analysis of Bar of Varying Sections:

Section 1

\[ P \leftarrow A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \rightarrow P \]

\[ L_1, L_2, L_3 \rightarrow \text{Length of Section 1, 2, and 3} \]

\[ A_1, A_2, A_3 \rightarrow \text{Cross-sectional area of Section 1, 2, and 3} \]

Then the total change in length of the bar is given by,

\[ \Delta L = P \left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3} \right) \]

An axial pull of 85000 N is acting on a bar consisting of three lengths as shown in fig.

If the Young's modulus = 2.1 x 10^5 N/mm^2. Determine

(i) Stress in each section and (ii) Total expansion of the bar

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (mm^2)</th>
<th>Stress (N/mm^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>85000/200</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>85000/300</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>85000/500</td>
</tr>
</tbody>
</table>

Total expansion = 85000 N
Stress in Section 1: \( \sigma_1 = \frac{35000}{\frac{\pi}{4} (20)^2} = 1111.408 \text{ N/mm}^2 \)

Stress in Section 2: \( \sigma_2 = \frac{35000}{\frac{\pi}{4} (30)^2} = 49.546 \text{ N/mm}^2 \)

Stress in Section 3: \( \sigma_3 = \frac{35000}{\frac{\pi}{4} (50)^2} = 17.825 \text{ N/mm}^2 \)

Total expansion:

\[
d_1 = \frac{P}{E} \left( \frac{A_1}{E_1} + \frac{A_2}{E_2} + \frac{A_3}{E_3} \right)
\]

\[
= \frac{35000}{2.1 \times 10^5} \left( \frac{200}{\frac{\pi}{4} (20)^2} + \frac{250}{\frac{\pi}{4} (30)^2} + \frac{220}{\frac{\pi}{4} (50)^2} \right)
\]

\[
= \frac{35000}{2.1 \times 10^5} \left( 6.366 + 3.536 + 1.120 \right)
\]

\[
d_1 = 0.183 \text{ mm}
\]
Principles of Superposition:

When a no. of loads are acting on a body, the resulting strain, according to the principle of superposition, will be the algebraic sum of strains caused by individual loads.

Prob. A brass bar, having cross-sectional area of 1000mm², is subjected to axial forces as shown.

A B C D

50kN

80kN 20kN

600mm 1m 1.20m

10kN

Find the total elongation of the bar. Take E = 1.05 x 10¹¹.

Soln.

The force of 80kN acting at B is split up into three forces of 50kN, 20kN & 10kN.

Then AB of the bar is subjected to a tensile load of 50kN.

50kN

50kN

50kN
Part BC is subjected to a compressive load of 20 kN.

Part BD is subjected to a compressive load of 10 kN.

Part AB

\[ \text{Increase in length of } AB = \frac{P_1}{AE} \times L_1 \]
\[ = \frac{50 \times 1000 \times 600}{1000 \times 1.05 \times 10^{-5}} = 0.2857 \text{ m} \]

Part BC

\[ = \frac{P_2}{AE} \times L_2 \Rightarrow \frac{20 \times 1000 \times 1000}{1000 \times 1.05 \times 10^{-5}} = 0.1904 \]

Part BD

\[ = \frac{P_3}{AE} \times L_3 \Rightarrow \frac{10 \times 1000 \times 2200}{1000 \times 1.05 \times 10^{-5}} = 0.2095 \]

Total elongation = 0.2857 - 0.1904 - 0.2095 = -0.1142 mm.
Analysis of Uniformly Tapering Circular Rod:

\[ d_L = \frac{4PL}{\pi E D^3} \quad \text{and} \quad \frac{4PL}{\pi E D_1 D_2} \]

Analysis of Uniformly Tapering Rectangular Bar:

\[ \text{Total extension: } \frac{PL}{E (a-b)} \log_e \frac{a}{b} \]

Analysis of Bars of Composite Section:

\[ f = \sigma_1 A_1 + \sigma_2 A_2 \]

\[ \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \]

\[ \frac{E_1}{E_2} \text{ is called modular ratio.} \]
A steel rod of 3 cm dia. is enclosed axially in a hollow copper tube of external dia 5 cm & internal dia 3 cm. The composite bar is then subjected to an axial pull of 25000 N. If the length of each bar is equal to 1.5 m, determine (i) The stresses in the rod & load carried by each bar. Take $E$ for steel = $2.1 \times 10^5$ N/mm$^2$ & for copper = $1.1 \times 10^5$ N/mm$^2$.

**Solution:**

**Dia of steel:**

**Area of steel rod $A_s$:**

$$A_s = \frac{\pi}{4} (3)^2 = 706.86 \text{mm}^2$$

**External area of copper tube:**

$$A_c = \frac{\pi}{4} [5^2 - 3^2] = 706.86 \text{mm}^2$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \Rightarrow \sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c = 1.909 \sigma_c$$

**Stress = Load / Area**

**Load = Stress / Area**

**Total load = Load on steel + Load on copper.**
\[ \sigma_3 \times A_3 + \sigma_c \times A_c = P. \]

\[ 1.909 \sigma_c \times 706.86 + \sigma_c \times 706.86 = 45000. \]

\[ \sigma_c \left(1.909 \times 706.86 + 706.86 \right) = 45000. \]

\[ 2056.25 \sigma_c = 45000. \]

\[ \sigma_c = \frac{45000}{2056.25} = 21.88 \text{ N/mm}^2. \]

Substitute the value of \( \sigma_c \) in eqn.

\[ \sigma_3 = 1.909 \times \sigma_c \]

\[ = 1.909 \times 21.88 \]

\[ = 41.77 \text{ N/mm}^2. \]

Load carried by each bar:

\[ \text{Load} = \text{Stress} \times \text{Area}. \]

\[ \therefore \text{Load carried by steel rod:} \]

\[ P_3 = \sigma_3 \times A_3. \]

\[ = 41.77 \times 706.86 = 29525.5 \text{ N.} \]

Load carried by copper tube:

\[ P_c = 45000 - 29525.5. \]

\[ = 15474.5 \text{ N.} \]
Thermal Stresses

Thermal stresses are the stresses induced in a body due to change in temperature.

Thermal strain, $\varepsilon = \frac{\text{Extension presented}}{\text{Original length}}$

$= \frac{dL}{L} = \alpha \cdot \Delta T$

$\alpha$ = Coefficient of linear expansion

$dL$ = Extension of rod due to rise of temp.

Thermal stress, $\sigma = \text{Thermal strain} \times E$

$= \alpha \cdot T \cdot E$

Thermal stress is also known as temperature stress.

Thermal stresses in composite bars:

$\sigma_1 + \frac{\sigma_2}{E_2} = \alpha_b \times T - \frac{\sigma_b}{E_b}$
Volumetric Strain:

\[ \varepsilon_v = \frac{\Delta V}{V} = 3 \varepsilon_v \text{ change in Volume} \]

\[ \varepsilon_v = \frac{\Delta V}{V} \text{ Original Volume} \]

Volumetric strain of a rectangular bar which is subjected to an axial load \( P \) in the direction of its length:

\[ \varepsilon_v = \frac{P}{E} \left( 1 - 2\nu \right) \]

2) Volumetric strain of a rectangular bar subject to three forces which are mutually perpendicular:

\[ \frac{\Delta V}{V} = \frac{1}{E} \left( \varepsilon_x + \varepsilon_y + \varepsilon_z \right) \left( 1 - 2\nu \right) \]

3) Volumetric strain on a cylindrical rod:

\[ \varepsilon_v = \frac{S_v - 2S_o}{S_o} \]

where \( \frac{S_v}{S_o} \) is the strain of length \( s \)

\[ \frac{S_v}{S_o} \] is the strain of diameter.
A metallic bar 300 mm x 100 mm x 40 mm is subjected to a force of 5 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y, and z directions. Determine the change in volume of the blocks. Take E = 2 x 10^5 N/mm², ν = 0.25.

\[ V = x \times y \times z = 12,000,000 \text{ mm}^3 \]

Load in x-direction = 5 kN = 5000 N.

Load in y-direction = 6 kN = 6000 N.

Load in z-direction = 4 kN = 4000 N.

Value of \( E = 2 \times 10^5 \text{ N/mm}^2 \)

\[ \nu = 0.25 \]

\( \sigma_x = \frac{5000 \text{ load in x-direction}}{x \times z} = \frac{5000}{300 \times 40} = 0.833 \text{ N/mm}^2 \)

\( \sigma_y = \frac{6000 \text{ load in y-direction}}{x \times z} = \frac{6000}{300 \times 40} = 0.5 \text{ N/mm}^2 \)

\( \sigma_z = \frac{4000 \text{ load in z-direction}}{x \times y} = \frac{4000}{100 \times 300} = 0.133 \text{ N/mm}^2 \)
\[ \frac{dv}{V} = \frac{1}{E} \left( \frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \right) (1 - 2\nu) \]

\[ = \frac{1}{2 \times 10^6} \left( 1.25 + 0.5 + 0.13 \right) (1 - 2\times 0.3) \]

\[ dv = \frac{1.883}{4 \times 10^5} \times 12,000,000 \]

\[ = 5.649 \text{ mm}^3 \]

**Bulk Modulus:**

\[ k = \frac{\text{Direct Stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\left(\frac{dv}{V}\right)} \]

**Stresses on Inclined Sections when the element is subjected to Simple Shear Stresses:**

![Diagram of inclined sections with shear forces](image)
Fig. shows a rectangular block ABCD which is in a state of simple shear & hence subjected to a set of shear stressess of intensity \( \tau \) on AB, CD, AD & CB.

Let the thickness of the block normal to the plane of the paper is unity.

It's required to find normal & tangential stresses across an inclined plane CE, which is having inclination \( \theta \) with the face CB.

Consider the equilibrium of the triangular piece CEB of thickness unity. The forces acting on the triangular piece CEB are shown in Fig.

(i) Shear force on face CB

\( Q_1 = \text{shear stress} \times \text{area of face CB} \)

\[ = \tau \times BC \times 1 \]

\[ = \tau \times BC \text{ acting along CB} \]

(ii) Shear force of face EB

\( Q_2 = \text{shear stress} \times \text{area of face EB} \)

\[ = \tau \times EB \times 1 = \tau \times EB \text{ acting along EB} \]
\[ P_n = Q_1 \sin \theta - Q_2 \cos \theta = 0 \]

\[ P_n = Q_1 \sin \theta + Q_2 \cos \theta \]

\[ = T \times Bc \times \sin \theta + T \times EB \times \cos \theta \]

\[ P_t = Q_1 \cos \theta + Q_2 \sin \theta = 0 \]

\[ P_t = Q_1 \cos \theta - Q_2 \sin \theta \]

\[ = T \times Bc \cos \theta - T \times EB \times \sin \theta \]

\[ \begin{aligned}
\sigma_n &= T \sin 2\theta \\
\sigma_t &= T \cos 2\theta 
\end{aligned} \]

**Principal Planes & Principal Stresses:**

The planes, which have no shear stress, are known as principal planes.
Once the principal planes are the planes of zero shear stress.

These planes carry only normal stresses.

The normal stresses acting on a principal plane are known as principal stresses.

\[
\text{Obliguity:}
\]

The angle made by the resultant given with normal to the oblique plane is known as obliquity. It's denoted by \( \phi \).

\[
\tan \phi = \frac{\varepsilon_y}{\varepsilon_x}
\]

Maximum Shear Stress:

The shear stress will be maximum when \( \sin 2\phi = 1 \) or \( 2\phi = 90^\circ \) or \( 180^\circ \).

\[
\phi = 45^\circ \text{ or } 135^\circ.
\]

And maximum shear stress \( (\tau_{\max}) = \frac{\sigma_t - \sigma_s}{2} \).

Principal Planes:

Moments normal to the

\[
\tau_{\max} = \frac{\sigma_t - \sigma_s}{2} \text{ at } \phi = \frac{\sigma_t + \sigma_s}{2}
\]
The resultant stresses:

$$\sigma_r = \sqrt{\sigma_1^2 + \sigma_2^2} \times \sin \theta$$

The stresses at a point in a bar are 200 N/mm² tensile & 100 N/mm² compressive. Determine the resultant stress in magnitude & direction on a plane inclined at 60° to the axis of major stress. Also determine the maximum intensity of shear stress in the material at point.

Major principal stress, $$\sigma_1 = 200 \text{ N/mm}^2$$

Minor principal stress, $$\sigma_2 = -100 \text{ N/mm}^2$$

Angle of the plane, which it makes with the major principal stress = 60°.

Angle = 90° - 60° = 30°
Resultant Stress in magnitude and direction:

\[ \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \]

\[ = 125 \text{ N/mm}^2 \]

\[ \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = 129.9 \text{ N/mm}^2 \]

Resultant:

\[ \sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{125^2 + 129.9^2} \]

\[ = 180.27 \text{ N/mm}^2 \]

\[ \tan \phi = \frac{\sigma_t}{\sigma_n} = 1.04 \]

\[ \phi = \tan^{-1} (1.04) = 46.6^\circ \]

Max. Shear Stress is given by:

\[ (\tau_{max}) = \frac{\sigma_1 - \sigma_2}{2} = 150 \text{ N/mm}^2 \]

A member subjected to Direct Stresses in two mutually \( \perp \) directions accompanied by Simple Bending Stress:

[Diagrams showing stress analysis]
\[ \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \]
\[ \sigma_c = \frac{\sigma_1 - \sigma_2}{2} \sin \theta - \cos \theta \]

For principal planes,
\[ \tan \beta = \frac{\sigma_c}{\sigma_m} \]

Major Principal Stress
\[ = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left( \frac{\sigma_1 - \sigma_2}{2} \right)^2 + \sigma_c^2} \]

Minor Principal Stress
\[ = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left( \frac{\sigma_1 - \sigma_2}{2} \right)^2 + \sigma_c^2} \]

Maximum Shear Stress
\[ \tan \gamma = \frac{\sigma_c - \sigma_2}{2 \sigma_m} \]

Max. Shear Stress
\[ \sigma_{max} = \frac{1}{2} \sqrt{(\sigma_m - \sigma_c)^2 + 4 \sigma_c^2} \]
Mohr’s Circle:

It's a graphical method of finding normal, tangential & resultant stresses on an oblique plane.

Mohr’s Circle can be drawn for the following cases:

(i) A body subjected to two mutually perpendicular tensile stresses of unequal intensities.

(ii) A body subjected to two mutually perpendicular principal stresses which are unequal & unlike (i.e., one is tensile & other is compressive).

(iii) A body subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress.