Unit - 2

Transverse loading on Beams & Stresses in

Shear Force & Bending Moment Diagrams:

* A shear force diagram is one which shows the variation of the shear force along the length of the beam.

* Bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

Types of Beams:

The following are the important types of beams:

1. Cantilever beam.
2. Simply supported beam
3. Overhanging beam

If the end portion of a beam is extended beyond support, such beam is known as overhanging beam.
4. Fixed beams:

A beam whose both ends are fixed or built in walls, is known as fixed beam.

5. Continuous beams:

A beam which is provided more than two supports.

Types of loads:

A beam is normally horizontal & the loads acting on beams are generally vertical. The following are the important types of load acting on a beam:

1. Concentrated or Point load.

2. Uniformly distributed load and

3. Uniformly varying load.

Concentrated or Point load:

A conc. load i.e., which is considered to act at a point,

\[ W \]
Uniformly Distributed Load:

- A uniformly distributed load is one which is spread over a beam in such a manner that the rate of loading \( w \) is uniform along the length.
- The rate of loading is expressed as \( w \) \( \text{N/m} \).
- It is represented as UDL.

For solving the numerical problems, the total UDL is converted into a pt. load acting at the Centre of Uniformly Distributed Load.

\[
\frac{w}{2}
\]

Uniformly Varying Load:

- A uniformly varying load is one which is spread over a beam in such a manner that the rate of loading varies from pt. to pt. along the beam in which load is zero at one end & increases uniformly to the other end. Such load is known as triangular load.
Shear Force & Bending Moment diagrams for a cantilever with a point load at the free end:

\[ F_x = + W \, . \]
\[ M_x = -Wx \, m \, . \]

Shear Force & Bending Moment Diagrams for a Cantilever with a Uniformly distributed load:

\[ W \, \text{unit length} \]
\[ \frac{W}{2} \, \text{at } A \]
\[ W \, \text{at } B \]
\[ M_x = \frac{Wx^2}{2} \, . \]
Mean Force & Bending Moment diagrams for a Cantilever carrying a gradually varying load.
A cantilever beam of length 2 m carries the point loads as shown in Figure. Draw the shear force & B.M. diagrams for the cantilever beam.

\[
\begin{align*}
A & \quad B \quad C \quad D \\
300N & \quad 500N & \quad 800N \\
0.5m & \quad 0.7m & \quad 0.8m
\end{align*}
\]

Shear Force Calculation:

\[
\begin{align*}
\text{S.F at D, } F_D &= 800 \text{ N} \\
\text{S.F at C, } F_C &= 800 + 500 = 1300 \text{ N} \\
\text{S.F at B, } F_B &= 800 + 500 + 300 = 1600 \text{ N} \\
\text{S.F at A, } F_A &= 1600 \text{ N}
\end{align*}
\]

Bending Moment Diagram:

The bending moment at D is zero.

(i) The bending moment at any section like C&D at a distance x & D is given by:

\[M_x = -800 \times x\]

B.M at C, \[M_C = -800 \times 0.8 = -640 \text{Nm}\]

B.M at B, \[M_B = -800 \times 1.5 - 500 (0.7) = -1550 \text{Nm}\]

B.M at A, \[M_A = -800 \times 2 - 500 (1.2) - 300 (0.5) - 800 (0.5) = -3600 \text{Nm}\]
Problem 2: A cantilever of length 2m carries a UDL of 2kn/m length over the whole length & a H.D. of 3kn at the free end. Draw the S.F. & B.M. diagrams for the cantilever.

Shear force at B = 3kn.

\( f_x = 3.0 + 2x \)
The above eqn. shows that shearing force follows a straight line.

At B, \( x = 0 \), hence \( F_B = 3 \text{ kN} \).

At \( x = 2 \text{ m} \), hence \( F_A = 3 + 2 \times 2 = 7 \text{ kN} \).

The Bending moment at any section at a distance \( x \) from the free end B is given by,

\[
M_x = - (3x + 2x^2 - \frac{x^2}{2})
\]

\[
= -(2x + \frac{2x^2}{2}) = 3x \left(1 - \frac{x}{2}\right).
\]

At B, \( x = 0 \). Hence, \( M_B = -(3 \times 0 + 0) = 0 \).

At A, \( x = 2 \text{ m} \). Hence, \( M_A = -(3 \times 2 + \frac{2}{2}) = -8 \text{ kNm} \).

\[ \frac{8 \text{ kNm}}{m} \]

\[ 8 \text{ kNm} \]
A cantilever beam of length 4m carries 0 kN/m at one end and 2 kN/m at the other end.

Draw the S.C & B.M. diagrams.

Shear force is zero at B. The shear force at C will be equal to the area of load diagram.

\[ \text{Shear force at C} = \frac{4 \times 2}{2} = 4 \text{ kN} \]

The B.M. at B is zero. The B.M. at A is equal to:

\[ MA = -\frac{Wl^2}{6} = -\frac{2 \times 4^2}{6} = -5.33 \text{ kNm} \]
Shear force & Bending Moment diagrams for a beam with a load at midpoint:

Shear force & B.m. Diagrams for a beam with eccentric forces.
S.F. & B.M. Diagrams for SSB Carrying UDL:

\[ \frac{W}{\text{unit length}} \]

\[ \frac{Wl}{2} \]

\[ \frac{Wl^2}{8} \]

S.F. & B.M. Diagrams for SSB Carrying UDL from zero at each end to \( \frac{W}{\text{unit length}} \) at the centre.

\[ \frac{Wl}{4} \]
A simply supported beam of length 6m carries the UDL & two pt. loads as shown in figure. Draw the S.F & B.M diagram for the beam. Also calculate the bending moment.
First calculate the reactions $R_A$ & $R_B$.

Taking moments of all forces about $A$, we get:

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40 \times 2 \times 4.$$  

$$= \quad 500. \quad \boxed{R_B = 50 \text{ kN}}.$$

$R_A = \text{Total Load on Beam} - R_B$.

$$= (50 + 10 \times 4 + 40) - 50 \Rightarrow 80 \text{ kN}.$$

**S.F. Diagram**

S.F. at $A$, $F_A = R_A = 80 \text{ kN}$.

S.F. just on right side of $C = R_A - 50 = 30 \text{ kN}$.

S.F. just on left side of $D = R_A - 50 - 10 \times 4 = -10 \text{ kN}$.

S.F. just on right side of $D = R_A - 50 - 10 \times 4 - 40 = -50$.

S.F. at $B = -50 \text{ kN}$.

**Now shear force at $E = R_A - 50 - 10(x - 2)$**

$$= 50 - 10x.$$

But shear force at $E = 0$:  

$$50 - 10x = 0; \quad x = 5 \text{ m}.$$

**B.M. Diagram**
B. M at C, \( M_c = R_a \times 2 = 160 \text{ kN.m} \)

B. M. at D, \( M_d = R_a \times 6 - 50 \times 4 - 10 \times 4 \times 4/2 \)

\[ = 200 \text{ kN.m} \]

At E, \( x = 5 \text{ m} \) & hence B. M at E.

\[ M_e = F_A \times 5 - 50 (5 - 2) - 10 (5 - 2) \times \left( \frac{5 - 2}{2} \right) \]

\[ = 205 \text{ kN/m} \]
S.F & B.M Diagrams for Overhanging Beams:

If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam.

- B.M is +ve below the two supports whereas B.M is -ve for overhanging portion.
- Hence at some pt., the B.M is zero after changing its sign from +ve to -ve.

Point of Contraluxure:

It's the pt. where B.M is zero after changing its sign from +ve to -ve or vice versa.

2 KN/m
When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam.

Due to the shear force and bending moment, the beam undergoes certain deformation.

The material of the beam will offer resistance or stresses against these deformations. These stresses, with certain assumptions, can be calculated.

The stresses introduced by bending moments are known as bending stresses or bending (or) simple bending.

\[ \begin{align*}
& \text{A} \\
& \text{B} \\
& \text{C} \\
& \text{D}
\end{align*} \]
Theory of simple bending with assumptions made:

1. The material of the beam is homogeneous and isotropic.

2. The value of Young's modulus of elasticity is the same in tension & compression.

3. The transverse sections which were plane before bending remain plane after bending also.

4. The beam is initially straight & all long filaments bend into circular area with a common center of curvature.

5. The radius of curvature is large compared with the dimensions of the cross-section.

6. Each layer of the beam is free to expand or contract, independently of the layer above or below.

Section Modulus:

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by symbol $W$. 
\[ Z = \frac{1}{y_{\text{max}}} \]

\[ f_{\text{m}} = \text{M.O.I about neutral axis} \]

\[ y_{\text{max}} = \text{Distance of the outermost layer from the neutral axis} \]

\[ \frac{M}{I} = \frac{f_{\text{max}}}{y_{\text{max}}} \]

\[ M = f_{\text{max}} \cdot I \frac{1}{y_{\text{max}}} \]

\[ M = f_{\text{max}} \cdot Z \]

A rectangular beam 200mm deep & 800mm wide is supported over a span of 8 m. What UDL / m the beam may safely carry, if the bending stress is not to exceed 120N/mm².

Given:

- Depth of beam \( d = 200\text{mm} \)
- Width of beam \( b = 800\text{mm} \)
- Length of beam \( L = 8\text{m} \)

Max. bending stress \( \sigma_{\text{max}} = 120\text{N/mm}^2 \)
\[ Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 2,000,000 \text{mm} \]

Max. B.M. for a s.s. B carrying UDL as shown:

\[ M = \frac{WL^2}{8} = \frac{W \times 8^2}{8} \Rightarrow 8W \text{ Nmm} \]

\[ = 8,000 \ W \text{ Nmm} \]

\[ M = 6 \text{max. } z \]

\[ 8000W = 120 \times 20,000,000 \]

\[ W = 30 \text{ kN/m} \]

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A rolled steel joist of I section has the dimensions as shown in Fig. This beam of I section carries a UDL of 40 kN/m run on a span of 10 m, calculate the max. stress produced due to bending.

**Given:**
- UDL: \( W = 40 \text{ kN/m} \)
- Span: \( l = 10 \text{ m} \)

Moment of inertia about the neutral axis:

\[ = \frac{200 \times 100^3}{12} - \frac{(200 - 10) \times 360^3}{12} \]

\[ = 3,279,465 \text{ mm}^4 \]
B.M is given by,

\[ M = \frac{Wc^2}{8} = \frac{20,000 \times 10^2}{8} = 5 \times 10^8 \text{Nmm} \]

Now using the relation,

\[ \frac{M}{I} = \frac{\sigma}{\gamma} \]

\[ \sigma = \frac{M}{I} \times \gamma \]

\[ \sigma_{\text{max}} = 304.92 \text{ N/mm}^2 \]

Shear Stresses in Beams:

The following are the important sections even which the shear stress distribution is to be obtained:

1. Rectangular Section.
2. Circular Section.
3. T-section.
4. T-sections and
5. Miscellaneous Sections.
A rectangular beam 100mm wide & 250mm deep is subjected to a maximum shear force of 50kN. Determine

(i) Average shear stress,

(ii) Maximum shear stress & (iii) shear stress at a distance of 25mm above the neutral axis.

**Given**

Width, $b = 100\text{mm}$

Depth, $d = 250\text{mm}$

Maximum shear force, $f = 50\text{kN} = 50,000\text{N}$

(i) Average shear stress is given by

$$\tau_{av} = \frac{f}{\text{Area}} = \frac{50,000}{100 \times 250} = \frac{50,000}{25,000} = 2\text{MPa}$$

(ii) Max. shear stress is given by

$$\tau_{\text{max}} = 1.5 \times \tau_{av} = 1.5 \times 2 = 3\text{N/mm}^2$$

(iii) The shear stress at a distance $y$ from $N$:

$$\tau = \frac{E}{2\gamma} \left( \frac{d^2}{4} - y^2 \right)$$
\[
\frac{50,000}{2 \frac{L}{2}} \left( \frac{2 \times 10^8}{L} - 2 \cdot 10^2 \right).
\]
\[
= \frac{50,000 \times 12}{2 \times 100 \times 250^2} \times 15,000 \text{ N/mm}^2.
\]
\[
= 2.89 \text{ N/mm}^2.
\]