\( \Delta u = u_2 - u_1 = mc_u (\tau_2 - \tau_1) \\
= 10 \times 0.6598 (323 - 298) \\
= 172.45 \text{ KJ} \)

\( \Delta h = mc_p (\tau_2 - \tau_1) \\
= 10 \times 0.9816 (323 - 298) \\
= 830.4 \text{ KJ} \)

\( \Delta s = mc_u \ln \left( \frac{\tau_2}{\tau_1} \right) + m R \ln \left( \frac{\tau_2}{\tau_1} \right) \)
Given Data:

\[ m = 10 \text{ kg} \]

\[ C_V_{N_2} = 0.745 \text{ kJ/kg.k} \]

\[ C_P_{N_2} = 1.041 \text{ kJ/kg.k} \]

\[ C_V_{CO_2} = 0.653 \text{ kJ/kg.k} \]

\[ C_P_{CO_2} = 0.842 \text{ kJ/kg.k} \]

\[ T_1 = 85^\circ C + 273 = 358K \]

\[ T_2 = 50 + 273 = 323K \]

To find:

For the mixture

\[ C_P = \frac{m_{N_2}C_P_{N_2} + m_{CO_2}C_P_{CO_2}}{m_{N_2} + m_{CO_2}} \]
\[ \text{volume of } \text{N}_2(\text{N}_2) = \frac{n \cdot \text{R} \cdot T}{P \cdot \text{N}_2} \]
\[\quad = 2.32 \text{m}^3\]

\[ \text{volume of mixture (V_m)} = V_{\text{O}_2} + V_{\text{N}_2} \]
\[\quad = 8.02 \text{m}^3\]

\[ \text{pressure of mixture (P_m)} = \frac{n_m \cdot \text{R} \cdot T}{V_m} \]
\[\quad = 114.5 \text{ kPa}\]

5. A perfect gas mixture consists of 4 kg of \text{N}_2 and 6 kg of \text{CO}_2 at a pressure of 4 bars and a temperature of 85°C.

\text{By \text{N}_2: CV = 0.745 \text{ KJ/kg.k} \text{ and CP = 1007.1 \text{ KJ/kg.k} \text{.}}
\[ \bar{m} = \frac{m_{\text{O}_2} \bar{v}_{\text{O}_2} + m_{\text{N}_2} + C_{\text{vN}_2} + T_{\text{N}_2}}{m_{\text{O}_2} C_{\text{vO}_2} + m_{\text{N}_2} C_{\text{vN}_2}} \]

\[ = \frac{(7 \times 0.658 \times 313) + (4 \times 0.743 \times 893)}{(7 \times 0.658) + (4 \times 0.743)} \]

\[ = 305.16 \text{ k} \]

Molecular weight of \( \text{O}_2 \), \( \mu_{\text{O}_2} = 2 \times 16 = 32 \text{ kg/kg-mol} \)

Molecular weight of \( \text{N}_2 \), \( \mu_{\text{N}_2} = 28 \text{ kg/kg-mol} \)

\[ \bar{N}_{\text{O}_2} = \frac{m_{\text{O}_2}}{\mu_{\text{O}_2}} = \frac{7}{32} = 0.219 \text{ kg/mol} \]
An insulated rigid tank is divided into two compartments by a partition. One compartment contains 7 kg of oxygen gas at 40°C and 100 kPa, and the other compartment contains 4 kg of nitrogen gas at 80°C and 150 kPa. Now the partition is removed, and the two gases are allowed to mix.

Determine:
1. The mixture temperature
2. The mixture pressure after equilibrium has been established

\[
\begin{align*}
\text{Given Data:} & \\
\text{m}_{O_2} &= 7 \text{ kg} \\
\end{align*}
\]

\[
\begin{align*}
\text{\(c_v\), \text{\(O_2\)} = 0.658 \text{ kJ/kg \cdot k} \\
\text{\(c_v\), \text{\(N_2\)} = 0.743 \text{ kJ/kg \cdot k} \\
\end{align*}
\]
Number of moles of mixture

\[ n_{\text{mix}} = \frac{n_P}{RT} \]

\[ = \frac{0.5 \times 1000}{8.314 \times 303} \]

\[ n_{\text{mix}} = 0.205 \text{ mol} \]

\[ (\text{i}) \]

Mass of mixture

\[ m_{\text{mix}} = n_{\text{mix}} \times m_m \]

\[ = 0.205 \times 38.4 \]

\[ = 7.881 \text{ kg} \]

\[ (\text{ii}) \]

Mass percentage

\[ \% = \frac{m_{\text{mix}}}{m_{\text{mix}} + m_{\text{air}}} \]

\[ = \frac{7.881}{7.881 + m_{\text{air}}} \]
A closed vessel has a capacity of 0.5 m$^3$. It contains 80% of N$_2$ and 80% O$_2$ 60% CO$_2$ by volume at 80°C and 1 atm. Calculate the molecular mass, gas constant, mass percentage and mass of mixture.

\[ M_{N_2} = 0.08 \text{ kg/ mol}. \]
\[ M_{O_2} = 0.04 \text{ kg/ mol}. \]
\[ M_{CO_2} = 0.03 \text{ kg/ mol}. \]

\[ \frac{R_{N_2}}{88} = 0.086 \text{ W/ kg. K}. \]
\[ \frac{R_{CO_2}}{88} = 0.018 \text{ W/ kg. K}. \]
\[ p_{CO_2} = \frac{m_{CO_2} R_{CO_2}}{V_m} \]

\[ = \frac{0.5 \times 0.189 \times 893}{0.2} \]

\[ = 138.49 \text{ kPa/m}^2 \]

Total pressure = 28.588 kPa

Molar mass of mixture
\[ \text{Molar mass of mixture} = \frac{\text{mass of mixture}}{\text{mole number of mixture}} \]

Mass of mixture = 4 + 1 + 0.5 = 5.5 kg

\[ \text{Number of moles of } N_2 = \frac{m_{N_2}}{M_{N_2}} = \frac{5.5}{0.028} \]

\[ = 0.1958 \text{ mol} \]
2. A tank contains 0.8 m³ of gas mixture composed of 4 kg of nitrogen, 1 kg of oxygen, and 0.5 kg of CO₂. If the temp is 50°C. Determine the total pressure, gas constant and molar mass of the mixture.

Solution

Molar mass of N₂

\[ \frac{8.314}{88} = \frac{R}{\mu_1} \]

\[ = 0.096 \text{ kJ/kg.k} \]

Molar mass of O₂

\[ \frac{8.314}{32} = 0.261 \text{ kJ/kg.k} \]

Molar mass of CO₂

\[ \frac{8.314}{44} = 0.190 \text{ kJ/kg.k} \]
\[ R = 2.5 \times 0.89694 + 4.5 \times 0.188 \]
\[ R = \frac{7}{7} \]
\[ R = 0.2375 \text{ kPa/l kg} \]

**Partial pressures and partial volume**

\[ P_{\text{N}_2} = x_{\text{N}_2} \times P = 0.46603 \times 400 = 186.432 \text{ kPa} \]

\[ P_{\text{CO}_2} = x_{\text{CO}_2} \times P = 0.53397 \times 400 = 213.556 \text{ kPa} \]

**Partial volume**

\[ PV = nRT \]

\[ V_{\text{N}_2} = \frac{P_{\text{N}_2} \times V_{\text{N}_2}}{P} \]
\[ x_{\text{CO}_2} = \frac{4.5}{44} = 0.1022 \]

\[ 2.5 + 4.5 = 7 \]

\[ 88 + 44 = 132 \]

2. **Equivalent molecular weight (\( \mu \))**

\[ \mu = x_N \mu_N + x_{\text{CO}_2} \mu_{\text{CO}_2} \]

\[ = 0.466 \times 88 + 0.533 \times 44 \]

\[ \mu = 36.54 \text{ kJ/kg mol} \]

3. **Equivalent gas constant (\( R \))**

\[ m = (18 \text{ g}) + (44 \text{ g}) = 62 \text{ g} \]

www.studentsfocus.com
1. A mixture of ideal gases consists of 2.5 kg of N₂ and 4.5 kg of CO₂ at a pressure of 4 bar and a temperature of 25°C. Determine

   1. Mole fraction of each constituent.
   2. Equivalent molecular weight of mixture.
   3. Equivalent gas constant of the mixture.
   4. Partial pressure and partial volume.
   5. Volume and density of the mixture.

Given Data:

\[ m_{N₂} = 2.5 \text{ kg} \quad m_{CO₂} = 4.5 \text{ kg} \]
Avogadro's Law:

Equal volume of different perfect gases at the same temperature and pressure contain equal number of molecules.

Characteristic gas equilibria

Generalized gas equation for ideal gas

\[
\frac{PV}{T} = n
\]

\[
\frac{PV}{T} = R
\]

\[
PV = nRT
\]
Charles's Law:

The volume of a given mass of a gas varies directly with its absolute temperature when the pressure remains constant.

\[ \frac{V_1}{V_2} = \frac{T_1}{T_2} \]  

The pressure of a given mass of a gas varies directly with its absolute temperature when the volume remains constant.

\[ \frac{P_1}{P_2} = \frac{T_1}{T_2} \]
Charles's Law:

The volume, $V$, of a given mass of a gas varies directly with its absolute temperature, $T$, when the pressure remains constant.

\[
\frac{V}{V_0} = c
\]

As long as pressure is constant,

The pressure, $P$, of a given mass of a gas varies directly with its absolute temperature when the volume remains constant.

\[
\frac{P}{P_0} = c
\]
Dividing by $\delta i$

\[
\left(\frac{\partial h}{\partial r}\right) \left(\frac{\partial p}{\partial \delta i}\right) + \left(\frac{\partial h}{\partial \rho}\right) = 0
\]

\[
\left(\frac{\partial h}{\partial r}\right) \left(\frac{\partial p}{\partial \delta i}\right) = -\left(\frac{\partial h}{\partial \rho}\right)_\rho
\]

\[
\left(\frac{\partial h}{\partial r}\right)_\rho \left(\frac{1}{\mu}\right) = -\left(\frac{\partial h}{\partial \rho}\right)_\rho
\]

\[
\left(\frac{\partial h}{\partial \rho}\right)_\rho = \frac{-1}{\mu} \left(\frac{\partial h}{\partial r}\right)_\rho
\]

\[
\left\{ \begin{array}{c}
C = \left(\frac{\partial h}{\partial \rho}\right)_\rho = \frac{-1}{\mu} \left(\frac{\partial h}{\partial r}\right)_\rho \\
\end{array} \right.
\]
Differentiating the equation with respect to pressure at constant enthropy:

\[ \left( \frac{\partial h}{\partial p} \right)_s = 0 + \frac{1}{\gamma \rho} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] \]

\[ \mu = \left( \frac{\partial h}{\partial p} \right)_s = \frac{1}{\gamma \rho} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] \]

Joule-Thomson coefficient for ideal gas.

Constant temperature coefficient:

\[ h = h(T) \]
Case (c): When $u$ is zero, the temp of the gas remains constant with throttling. The temperature at which $u = 0$ is called inversion temperature for a given pressure.

Inversion curve:

The maximum point on each curve is called inversion point, and the locus of the inversion point is called inversion curve.

\[ \Delta h = \left[ Cp di - i \left( \frac{\partial v}{\partial T} \right) dp \right] + T dp \]

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\[
\mu = \left( \frac{\partial i}{\partial p} \right)_h
\]

\[
\left( \frac{\partial f}{\partial p} \right)_h = 0
\]

- Cooling zone
- Blasting zone
- Inversion point
- Inversion casing

Case (c)

\[
\mu = \left( \frac{\partial i}{\partial p} \right)
\]

Presurization & Temperature Changes are negative.
Fig. shows the arrangement of porous plug experiment. In this experiment, a stream of gas at a pressure \( P_1 \) and temperature \( T_1 \) is allowed to flow continuously through a porous plug. The gas comes out from the other end of the porous plug at a pressure \( P_2 \) and temperature \( T_2 \).

From SFEE

\[
g_{z_1} + \frac{v^2}{2} + h_1 + \phi_e = g_{z_2} + \frac{v^2}{2} + h_2 + \phi_f
\]
Joule-Thomson coefficient:

It is defined as the change in temperature with change in pressure keeping the enthalpy remains constant. It is denoted by \( u \).

\[ u = \frac{\partial T}{\partial P} \]

Throttling process:

It is defined as the fluid expansion through a minute orifice (or slightly opened valve). During the throttling process, pressure and velocity are reduced. But there is no heat transfer and no work done by the system during this process.
\[ \int_{s_f}^{s_g} ds = \frac{dP}{dV} \int_{V_f}^{V_g} dv \]

\[ [S]_{s_f}^{s_g} = \frac{dP}{dV} [V]_{V_f}^{V_g} \]

\[ [s_g - s_f] = \frac{dP}{dV} [v_g - v_f] = \frac{s_g - s_f}{v_g - v_f} \]

\[ \frac{dP}{dV} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}} \quad \rightarrow (b) \]

From the second law of thermodynamics.

\[ ds = \frac{dg}{v_{fg}} \]

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when the phase is changing from saturated liquid to saturated vapour, temperature remains constant, so ds equation reduces to

\[ ds = \left( \frac{\partial s}{\partial u} \right)_v \, du \]

\[ \Rightarrow \int_1^2 0 \]

from overall equation
Sub. 8 and 9 in eq. 7

\[ c_p - c_v = \frac{1 + \beta^2 u^4}{\alpha y} \]

\[ c_p - c_v = \frac{v + \beta^2 \gamma}{\alpha} \]

Clapeyron Equation

Clapeyron equation which involves the relationship between the saturation pressure, saturation temperature, the enthalpy of evaporation, and the specific volume of the two phases involved. This equation provides a basis for calculating it gives...
From the above, we conclude:

$$C_p - C_v = \frac{1}{\frac{dp}{dT}} u \cdot \left( \frac{du}{dT} \right)_p \rightarrow 5$$

We know that:

$$\frac{(dp)}{(dv)} + \frac{(du)}{(dT)} \cdot \frac{(dT)}{dp} \cdot (\frac{dt}{dp}) = -1$$

$$\frac{(dp)}{(dT)} \cdot \frac{(dt)}{(du)} \cdot \frac{(dv)}{(dp)} = -1$$

$$\frac{(dp)}{(dT)} u = -\frac{(dv)}{(dT)} \cdot \left( \frac{dp}{dv} \right)_T \rightarrow 6$$

Sub eq. 6 in 5:

$$C_p - C_v = \frac{1}{\frac{dp}{dT}} u \cdot \left( \frac{du}{dT} \right)_T \left( \frac{du}{dT} \right)_p$$
\[
\left( \frac{\partial p}{\partial t} \right)_u \, du + \left( \frac{\partial v}{\partial t} \right)_p \, dp = \frac{c_p}{t} \, d\bar{t} - \frac{c_v}{t} \, d\bar{t}
\]

\[
-t \left[ \left( \frac{\partial p}{\partial t} \right)_u \, du + \left( \frac{\partial v}{\partial t} \right)_p \, dp \right] = (c_p - c_v) \, d\bar{t}
\]

\[
\left[ \frac{\int \left( \frac{\partial p}{\partial t} \right)_u \, du}{c_p - c_v} \right] \, dv + \left[ \frac{\int \left( \frac{\partial v}{\partial t} \right)_p \, dp}{c_p - c_v} \right] \, dp = d\bar{t}
\]

Choose a function \( \bar{t} = \bar{t}(u, p) \) → (3)

\[
d\bar{t} = \left( \frac{\partial \bar{t}}{\partial u} \right)_p \, du + \left( \frac{\partial \bar{t}}{\partial p} \right)_u \, dp \]

explanation of eq. (3) & (4)
According to Maxwell's equation

\[
\frac{\partial s}{\partial p} = -\left(\frac{\partial U}{\partial T}\right)_p \rightarrow ③
\]

from the enthalpy changes

\[
c_p = \frac{1}{\gamma} \left(\frac{\partial s}{\partial T}\right)_p \rightarrow ③
\]

Substitute ② & ③ in eq ①

\[
ds = \frac{c_p}{\gamma} \, dT - \left(\frac{\partial U}{\partial T}\right)_p \, dp
\]

Generalised relation for specific heat \(c_p\) & \(c_v\)

we know that
Entropy change: \((ds)\)

Case (c)

Take a function \(S = S(i, u)\)

\[ ds = \left(\frac{\partial S}{\partial i}\right)_u di + \left(\frac{\partial S}{\partial u}\right)_i du \rightarrow (1) \]

According to Maxwell relation

\[ \frac{\partial S}{\partial u} = \frac{\partial P}{\partial i} \rightarrow (2) \]

From the internal energy change equation

\[ C_U = T \left(\frac{\partial S}{\partial i}\right)_u \]

\[ C_V = \left(\frac{\partial S}{\partial i}\right)_u \rightarrow (3) \]

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Now compose the co-efficient of eqn 3 & 6

\[ c_p = \frac{1}{\left( \frac{\partial s}{\partial t} \right)_p} \]

\[ \left( \frac{\partial h}{\partial p} \right)_T = \left( \frac{\partial s}{\partial p} + u \right) \]

→ 7

According to Maxwell relabing we have

\[ \left( \frac{\partial s}{\partial p} \right)_T = - \left( \frac{\partial v}{\partial t} \right)_p \]

→ 8

Sub 7 & 8 in eqn 3

\[ dh = c_p dT + \left( \frac{\partial s}{\partial p} \right)_T + u \frac{dp}{p} \]
Enthalpy changes: \((dh)\)

Take a function \(h = h(T, P)\)

\[
dh = \left(\frac{\partial h}{\partial T}\right)_P \, dT + \left(\frac{\partial h}{\partial P}\right)_T \, dP \rightarrow 0
\]

Specific heat at constant pressure

\[C_P = \left(\frac{\partial h}{\partial T}\right)_P \rightarrow 2\]

Sub eq 2 in eq 1

\[
dh = C_P \, dT + \left(\frac{\partial h}{\partial P}\right)_T \, dP \rightarrow 3
\]

Choose a function

\[S = s(T, P)\]
\[
e^T \left( \frac{\partial s}{\partial T} \right)_u \, dt + \left[ e^T \frac{\partial s}{\partial u} \right] \, du \rightarrow 0
\]

Now compare the co-efficient of \( du \) \( \Rightarrow \) \( \Rightarrow \)

\[
c = e^T \frac{\partial s}{\partial T} \frac{du}{dt} = \left( \frac{\partial s}{\partial u} \right)_t \rightarrow 0
\]

According to Maxwell relation we know that

\[
\left( \frac{\partial s}{\partial u} \right)_t = \left( \frac{\partial p}{\partial T} \right)_u \rightarrow 8
\]

Sub \( \Rightarrow \) \( \Rightarrow \) \( \Rightarrow \) \( \Rightarrow \)
General relation of $du$, $dh$, $ds$ and $cp$ & $cv$

1. Internal energy change: $(du)$

Take a function $U = U(i, v)$

$$du = \left(\frac{\partial u}{\partial i}\right)_v di + \left(\frac{\partial u}{\partial v}\right)_i dv$$

Specific heat at constant volume $c_v \rightarrow 1$

Specific heat at constant pressure $c_p \rightarrow 2$

Sub eq $(2)$ in eq $(1)$

$$du = c_v di + \left(\frac{\partial u}{\partial v}\right)_i dv \rightarrow 3$$

Choose a function

So $c(v, u)$
\[ \mathbf{w} \cdot \mathbf{u} = \mathbf{d} \cdot \mathbf{s} - p \mathbf{d} \mathbf{u} \rightarrow 6 \]

Substitute \( \mathbf{u} \) in Eq. 5

\[ d\mathbf{u} = \int \left[ \frac{d\mathbf{s}}{dt} \right] \mathbf{u} \, dt + \int \left( \frac{d\mathbf{s}}{d\mathbf{u}} \right) \mathbf{u} \cdot d\mathbf{u} - p \mathbf{d} \mathbf{u} \]

\[ = \int \left( \frac{d\mathbf{s}}{dt} \right) \mathbf{u} \, dt + \left[ \frac{d\mathbf{s}}{d\mathbf{u}} \right] \mathbf{u} \cdot d\mathbf{u} - p \mathbf{d} \mathbf{u} \rightarrow 6 \]

Now compare the coefficient of Eq. 3 & 6

\[ c\mathbf{u} = \int \left( \frac{d\mathbf{s}}{dt} \right) \mathbf{u} \]

\[ \left( \frac{d\mathbf{u}}{d\mathbf{u}} \right) \mathbf{u} = \int \left( \frac{d\mathbf{s}}{d\mathbf{u}} \right) \mathbf{u} \cdot d\mathbf{u} - p \rightarrow 7 \]

According to Maxwell relation \( \mathbf{u} \cdot \mathbf{u} \cdot \mathbf{u} \)

\[ \left( \frac{d\mathbf{s}}{d\mathbf{u}} \right) \mathbf{u} = \left( \frac{d\mathbf{p}}{d\mathbf{u}} \right) \mathbf{u} \rightarrow 8 \]
\[
\frac{\partial v}{\partial s} = \frac{\partial v}{\partial P} \quad \frac{\partial s}{\partial P} = -\frac{1}{\partial v} \quad \frac{\partial s}{\partial v} = \frac{\partial P}{\partial P}
\]

General relations of \(du, dh, ds\) and \(cp \& cu\)

Internal energy changes: \(\Delta u\)

Take a function \(U = u(t, u)\)
By Gibbs Function

\[ G_1 = h - TS \]
\[ dg = dh - d(TS) \]
\[ dg = dh - Tds - SdT \rightarrow 6 \]

Substitute \( dh \) value in equation 6

\[ dg = -\dot{V}dt + \int ds + \int Fds + \int Vdp \]
\[ dg = \frac{\partial V}{\partial P} - SdT \rightarrow 7 \]

\[ d\mu = \int ds - Pdv \]
\[ dh = Tds + Vdp \]
\[ da = -Pdv - SdT \]
\[ dg = Vdp - SdT \]

1 \rightarrow \left( \frac{\partial T}{\partial S} \right)_P = -\left( \frac{\partial P}{\partial S} \right)_T \rightarrow 8 \]
Maxwell's Equations

Maxwell's equations relate entropy to the three directly measurable properties $P, V,$ and $T$ for a pure simple compressible substance.

From the laws:

$A = W + SU$

Re-arranging the parameters:

$\frac{A}{S} = x + W$

$\frac{dA}{dS} = dx + dW$

$\Rightarrow \frac{dA}{dS} = dx - pdV$

With $x$: $dx = \frac{dA}{dS}$

$W = pdV$

$h = u + pv$

$\Delta h = du + d(PV)$

$\Delta h = 0$

Value in eq.