Unit 1

Introduction:

- Communication medium => Wire Conducting, Cable, or Free Space
  - New medium is => Optic Cable (Fibre)

→ They carry the information in form of light beam from one place to another.

→ The freq of optical spectrum: $3 \times 10^{14}$ Hz

→ Fiber optic cable
  - Information carrying capability
  - wider bandwidth
  - Tx capacity of optic fiber link: $B \times L$

  $B$ => Tx bit rate

  $L$ => Repeater spacing

→ Lower loss => less signal attenuation over long distance

→ Light weight => lighter than copper cables

→ Security => cannot be 'eavesdropped' easily.
* Without repeaters, we can Tx with max data rate of 56b/s over a distance of 111km.

* Low cost & maintenance

Fiber optics technology involves the emission, transmission, and detection of light.

* Element of optical fiber Tx link:
  a) Light source
  b) Light signal Tx
  c) Optical fiber
  d) Photodetecting receiver

Fiber & cable splices
Connectors
Receivers
Beam splitters
Optical amplifiers

a) Reflection:

\[ \theta_1 = \theta_2 \]

\( \theta_1 \) = angle of incidence
\( \theta_2 \) = angle of reflection

\( \theta_1 = \theta_2 \) Law of reflection

b) Refraction:

b) Refraction:

\[ \text{Bending of light} \]

Refraction Index: \( n = \text{Amount of refraction} \)

\[ n = \frac{\text{Speed of light in air}}{\text{Speed of light in substance}} \]
Values of $n$ (refractive index) 
- Air = 1.00
- Water = 1.33
- Olive glass = 1.45
- Diamond = 2.42

Snell's Law:

$\theta_1 \sin \theta_1 = n_2 \sin \theta_2$

$n_1 > n_2$

Snell's law

Critical angle: $\theta_c$ angle of incidence that causes the refracted light to travel along the interface below two different media.

Using Snell's Law

$\sin \theta_1 = \frac{n_2}{n_1} \sin \theta_2$

with $\theta_2 = 90^\circ$, $\theta_1$ becomes $\theta_c$ Critical angle

$\sin \theta_c = \frac{n_2}{n_1}$

$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$

Total internal reflection:

$\theta_1$ when angle of incidence $\theta_1$ is greater than $\theta_c$

Condition of TIR is satisfied.

Then total light will be reflected back $\theta_1 = \theta_2$
\[ \phi_i < \phi_c \]

\[ \phi_i = \phi_c \]

\[ \phi_i > \phi_c \]

\[ \Rightarrow \text{it occurs only in material in velocity of light slower than air.} \]

* Numerical Aperture (NA)

\[ NA = \sin \theta_{in} \]

\( \theta_{in} \rightarrow \text{Acceptance angle (degrees)} \)

* Acceptance angle:

\( \theta_{in} \rightarrow \text{max angle to the fiber axis at which light may enter the fiber axis in order to be propagated.} \)

\[ \theta_{in(max)} = \sin^{-1} \left( \frac{\sqrt{n_f^2 - n_i^2}}{n_o} \right) \]

\( n_o = 1 \)

\[ \theta_{in(max)} = \sin^{-1} \left( \frac{\sqrt{n_f^2 - n_i^2}}{n_i} \right) \]

\( \frac{n_f}{n_i} = 1 \)

\[ \frac{n_f}{n_i} = \sqrt{n_f^2 - n_i^2} \]

\[ \left( n_f = n_i \right) \]

\[ \delta n_i = n_i - n_e \]

\[ \Delta n_i = n_i - n_e \]

\[ \Delta = n_i^2 - n_e^2 \]

\[ \frac{\Delta n_i}{2n_i} = \frac{n_i - n_e}{n_i} \]

\[ \Delta n_i = n_i - n_e \]

\[ \frac{n_i^2 - n_e^2}{2n_i} \]
Types of rays:

- Meridional ray
- Skew ray
- Bound ray
- Unbound ray

Ray optics representation of skew rays:

The angle of acceptance is known as maximum possible angle of launching of a light ray that is accepted by the fiber.

Numerical Aperture: \( (NA) \)

By Snell's Law

\[
\sin \alpha = \frac{n_1 \sin \theta}{n_2} \tag{1}
\]

For figure \( \theta = \frac{\pi}{2} - \phi \)

Sub \( \phi \) in \( \text{eqn}(1) \)

Basic trigonometric ratio:

\[
\cos \theta_c = \sqrt{1 - \sin^2 \theta_c} \]

Apply Snell's Law

we get,

\[
\sin \theta_c = \frac{n_2}{n_1} \tag{2}
\]

\[
\cos \theta_c = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \tag{3}
\]

Sub \( \cos \theta_c \) in \( \text{eqn}(4) \)

\[
\sin \alpha = \sqrt{n_1^2 - n_2^2} \tag{4}
\]

So by Snell's Law

\[
\sin \alpha = n_1 \sin \theta \tag{1}
\]

For figure \( \theta = \frac{\pi}{2} - \phi \)

Sub \( \phi \) in \( \text{eqn}(1) \)
Skew rays:

- Skew rays are not through the fiber axis.
- It follows a helical path in a fiber.
- They will not lie in a single plane, more difficult to track.

* great power loss arises when skew rays are included in the analyses, they are trapped in the fiber and actually become rays.

\[ n_0 \rightarrow \text{it get attenuates as the light travels along the optical waveguide.} \]

To find ray path AB:

* Acceptance condition

\[ n_0 \sin \theta \cos \gamma = (n_1^2 - n_2^2)^{1/2} \cdot NA \]

Electromagnetic wave Theory:

a) EM waves:

- It is to obtain improved model for the propagation of light in fiber.
- Basic is from Maxwell's Equation.
Curl Equation:
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \frac{\mathbf{J}}{\varepsilon_0} \]

Divergence Condition:
\[ \nabla \cdot \mathbf{D} = \rho \] [no free charges]
\[ \nabla \cdot \mathbf{B} = 0 \] [no free poles]

Relationship of \( \mathbf{D}, \mathbf{B}, \mathbf{E}, \mathbf{H} \):
\[ \mathbf{D} = \varepsilon_0 \mathbf{E} \] (i)
\[ \mathbf{B} = \mu_0 \mathbf{H} \] (ii)
\[ \varepsilon \rightarrow \text{dielectric permittivity} \]
\[ \mu \rightarrow \text{magnetic permeability of the medium} \]

Sub for \( \mathbf{D} + \mathbf{B} \):
\[ \nabla \times (\nabla \times \mathbf{E}) = \mu_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{H}) = -\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \]
\[ \nabla \times (\nabla \times \mathbf{H}) = -\varepsilon_0 \frac{\partial \mathbf{B}}{\partial t} \]

Phase velocity in dielectric medium:
\[ v_p = \frac{1}{\mu_0 \varepsilon_0 \sqrt{\mu_0 \varepsilon_0}} = \frac{1}{(\mu_0 \varepsilon_0 \varepsilon_\mu \varepsilon_0)^{1/2}} \]

Velocity of free space:
\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]
* Modes in a planar guide:

A planar guide simple form of an optical waveguide.

\[ \beta_z = n_z \frac{\omega}{c} \]

\[ \beta_x = n_x k \sin \theta \]

\[ n_z > n_x \Rightarrow \beta_z = n_z k \cos \theta \]

Ray propagation in a plane dielectric guide.

* Phase velocity: \( v_p \)

A monochromatic light wave propagates along a guide in the \( z \) direction; these points of constant phase travel at a phase velocity \( v_p \):

\[ v_p = \frac{\omega}{k} \]

\[ \beta = \frac{2\pi}{\lambda} \]

is given by

\[ \lambda = \frac{\omega}{k} \]

\( \omega = \) angular frequency

\( k = \) propagation constant
Group velocity ($V_g$) = \frac{\partial \omega}{\partial k}

$\Rightarrow$ group of waves with similar frequencies propagate

$\Rightarrow$ This does not travel in $V_p$

$\Rightarrow$ but in group velocity ($V_g$)

Envelope of wave package ($V_g$).

Propagation constant ($\beta$)

$\beta = \eta \cdot \frac{2\pi}{\lambda}$

Consider \eta \rightarrow infinite medium of refractive index

$V_p = \frac{\omega}{k} = \frac{\omega}{n, \omega \frac{c}{\omega}} = \omega \times \frac{c}{n, \omega}$

$V_p = \frac{c}{n_1}$

group velocity $V_g = \frac{d\omega}{dp} \cdot \frac{dx}{dx}$

$V_g = \frac{c}{V_g}$

$\Rightarrow$ group index of guide.
Single Mode Fibers:

designed to allow only one mode of propagation.

- Diameter of fiber 8-12 μm

- Small index difference between the core and cladding with freq V ~ 2.4

LP01 mode propagation possible over the range 0 ≤ V ≤ 2.405

Mode Field Diameter:

- Take into account the V dependent field penetration into the fibre cladding

* Field distribution across the Air Fiber

\[ E(r) = E_0 \exp \left( -\frac{r^2}{W_0^2} \right) \]

- \( r \) → radius of field distribution

- \( E_0 \) → field at zero radius

- \( W_0 \) → Width of Optical Field Distribution

Propagation modes in Sym Fibers:

- Two independent degenerate modes

Propagate within the Sym Fibers. They are similar

Polarization Planes are orthogonal.
Birefringence:

Polarization modes propagate with different phase velocities, and the difference is their effective refractive indices called Birefringence.

\[ B_p = n_y - n_x \]

\[ \beta = k_0 (n_y - n_x) \]  \( \rightarrow \)  \[ k_0 = \frac{2\pi}{\lambda} \]  \( \text{Free Space Propagation} \)

Fiber beat length:

\[ L_p = \frac{2\pi}{\beta} \]

Sub equation in 2:

\[ L_p = \frac{\lambda}{n_y - n_x} \]

\[ L_p = \frac{\lambda}{B_p} \]
Cylindrical wave guide:

- Has 2 dimensions: TE_{lm} + TM_{lm} nodes for hybrid modes: \( \text{HE}_{lm} \cdot \text{EH}_{lm} \)

Scalar wave equation:

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} + \left( \mu^2 \kappa^2 - \beta^2 \right) \psi = 0
\]

\( \psi \) \( \rightarrow \) field (\( E \) or \( H \))

\( \mu, n \) \( \rightarrow \) refractive index of core

\( \beta \) \( \rightarrow \) propagation constant

\( r, \phi \) \( \rightarrow \) cylindrical coordinates

\( \rightarrow \) \( \beta \) lies in range:

\( n_1 \kappa < \beta < n_2 \kappa \)

\( n_0 \) \( \rightarrow \) refractive index of cladding.

\( \text{LP}_{01}, \text{HE}_{11} \)

\( \text{TE}_{11}, \text{HE}_{11} \)

\( \text{LP}_{11}, \text{TM}_{01} \)

\( \text{HE}_{21} \)
$U, W$ are the eigen values in core + cladding

$$U = a \left( n_1^2 k^2 - \beta^2 \right)^{1/2}$$

$$W = a \left( \beta^2 - n_2^2 k^2 \right)^{1/2}$$

Normalized frequency ($V$)

$$V = (U^2 + W^2)^{1/2}$$

$$V = a k \left[ n_1^2 - n_2^2 \right]^{1/2}$$

$$V = \frac{2 \pi}{\lambda} a (n_1^2)$$

$$V = \frac{2 \pi}{\lambda} a n_1 \left( 2 \Delta \right)^{1/2}$$

Normalized propagation constant ($b$)

$$b = 1 - \frac{U^2}{V^2} \approx \frac{(B/k)^2 - n_2^2}{n_3^2 - n_2^2}$$

$$b = \frac{(B/k)^2 - n_2^2}{2 n_1^2 \Delta}$$
Weak Guidance Approximation:

Field matching condition at boundary

\[
\frac{U J_{k_{\text{eff}}} (u)}{J_{1} (u)} = i \omega \frac{K_{k_{\text{eff}}} (u)}{k_{1} (u)}
\]

Leaky modes:

- the fields are confined partially in the fiber core and attenuated as they propagate along fiber length due to radiation + tunnel effect.
Introduction of Wave model

- Breaking light as transverse electromagnetic wave

* Different TE modes in an optical fiber:

\[ \frac{2 \pi \sin \theta}{\lambda} + \frac{\gamma}{\lambda} = m \]  \( (m = 0, 1, 2, 3, \ldots) \)

* Graded Index Optical fiber:

* Cylindrical Co-ordinate System:

\( (r, \phi, z) \)  \( \rightarrow n_z \)

\( r \rightarrow \text{Radial distance of the point from axis of fiber} \)

\( \phi \rightarrow \text{Angle b/w plane contain pt and reference plane} \)
* Basic Wave Equation *

Dielectric constant of 
the core, \( \varepsilon_1 = \varepsilon_0 n_1^2 \)

Dielectric constant of 
the cladding \( \varepsilon_2 = \varepsilon_0 n_2^2 \)

Maxwell's equation for electric + magnetic

\[ \nabla \cdot \mathbf{D} = 0 \quad - (a) \quad \mathbf{D} \quad \text{Electric displacement vector} \]

\[ \nabla \cdot \mathbf{B} = 0 \quad - (b) \quad \mathbf{B} \quad \text{Magnetic flux density} \]

\[ \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad - (c) \quad \mathbf{B} = \mu_0 \mathbf{H} \]

\[ \nabla \times \mathbf{H} = - \frac{\partial \mathbf{D}}{\partial t} \quad - (d) \quad \mathbf{D} = \varepsilon_0 \mathbf{E} \]

**Step 1** Sub \( \mathbf{D} = \varepsilon_0 \mathbf{E} \) in eq (a) [de-coupling]

\[ \nabla \cdot (\varepsilon_0 \mathbf{E}) = 0 \]

\[ \nabla \cdot \mathbf{E} = 0 \quad [\varepsilon \text{ is independent of space}] \]

From eqn (b)

\[ \nabla \cdot \mathbf{H} = 0 \quad \text{Since fiber is dielectric} \]

**Step 2** \( \rightarrow \) Curl of each equation + Sub each eqn with other

Curl of eqn (c)

\[ \nabla \times \nabla \times \mathbf{E} = - \nabla \times \frac{\partial \mathbf{D}}{\partial t} \]

\[ \nabla \times \nabla \times \mathbf{E} = - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

Sub \( \nabla \times \mathbf{E} \) from eqn (c)

\[ \nabla^2 \mathbf{E} = \varepsilon_0 \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \]

Add for eqn (d)

\[ \nabla \times \mathbf{H} = \mu_0 \mathbf{E} \frac{\partial^2 \mathbf{H}}{\partial t^2} \]
Waveguide Equation:

Cylindrical co-ordinate system \( r, \phi, z \): 

\[ \text{wave are to propagate along the } Z \text{-axis} \]

Functional dependence: 

\[ E = E_0 (r, \phi) e^{j(\omega t - kz)} \quad (1) \]

\[ H = H_0 (r, \phi) e^{j(\omega t - kz)} \quad (2) \]

Equations 1 and 2 are due to Maxwell Equations.

Step 1

\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad (6) \]

\[ \frac{1}{r} \left( \frac{\partial}{\partial \rho} \left( \rho E_\rho \right) - \frac{1}{r} \frac{\partial}{\partial \phi} \left( \rho E_\phi \right) \right) = -j \omega \mu H_\phi \quad (4) \]

\[ \nabla \times H = \frac{\partial D}{\partial t} \quad (6) \]

\[ \frac{1}{r} \left( \frac{\partial}{\partial \rho} \left( \rho H_\rho \right) - \frac{1}{r} \frac{\partial}{\partial \phi} \left( \rho H_\phi \right) \right) = -j \omega \varepsilon E_\phi \quad (4) \]

Step 2

From equations \( \nabla \times E = \frac{\partial D}{\partial t} \)

\[ \frac{1}{r} \left( \frac{\partial}{\partial \rho} \left( \rho E_\rho \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \rho E_\phi \right) \right) = j \omega \mu H_\phi \quad (6) \]

\[ \frac{1}{r} \left( \frac{\partial}{\partial \rho} \left( \rho H_\rho \right) - \frac{1}{r} \frac{\partial}{\partial \phi} \left( \rho H_\phi \right) \right) = j \omega \varepsilon E_\phi \quad (6) \]

Step 3

\[ E_\rho = -\frac{i}{q_e} \left( \frac{\beta}{r} \frac{\partial E_\phi}{\partial r} + \frac{\mu \omega}{r} \frac{\partial H_\rho}{\partial \phi} \right) \quad (3) \]

\[ E_\phi = -\frac{i}{q_e} \left( \frac{\beta}{r} \frac{\partial E_\rho}{\partial \phi} - \frac{\mu \omega}{r} \frac{\partial H_\rho}{\partial r} \right) \quad (4) \]
\[ A_y = \frac{-i}{q^2} \left( \frac{\partial}{} \frac{\partial E_z}{\partial y} \right) \]

\[ A_y = \frac{-\frac{\partial}{\partial y} \left( \frac{\partial B_r}{\partial x} - \frac{\partial B_z}{\partial y} \right)}{q^2} \left( \frac{\partial^2 B_z}{\partial x^2} - \frac{\partial B_x}{\partial y} \right) \]

\[ H_0 = -\frac{i}{q^2} \left( \frac{\partial H_z}{\partial y} + \omega e \frac{\partial E_z}{\partial y} \right) \]

\[ q^2 = \omega^2 \mu_0 - \beta^2 = k^2 - \beta^2 \]

**Step 4** \( \Rightarrow \) Sub 5 + 6 in equation (C)

\[ \frac{\partial^2 E_z}{\partial y^2} + \frac{1}{r} \frac{\partial E_z}{\partial y} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0 \]

**Step 5** \( \Rightarrow \) Sub 3 + 4 in equation (C)

\[ \frac{\partial^2 H_z}{\partial y^2} + \frac{1}{r} \frac{\partial H_z}{\partial y} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} + q^2 H_z = 0 \]

- x -

if \( E_z = 0 \) \( \Rightarrow \) modes are called TE

\( H_z = 0 \) \( \Rightarrow \) TE \( \Rightarrow \) TH

\( E_z + H_z \) is non-zero \( \Rightarrow \) hybrid mode

\( H_z \Rightarrow HE \)

\( E_z \Rightarrow EH \)

\( \rightarrow \text{hybrid mode} \)
Single-mode fibers:

- Allow only one mode to propagate
  - Other modes will be absorbed
- By reducing core-diameter
- Core-diameter will be selected according as V-number is less than 2.4
- Index difference will be small.

a) Mode Field Diameter: \[ [\text{MFD}] \]

- Geometric distribution of light in the propagation mode.

\[ \text{MFD} = 2W_0 = 2 \left[ \sqrt{\frac{\int_0^\infty E^2(y) y^2 dy}{\int_0^\infty E^2(y) y dy}} \right]^\frac{1}{2} \]

\[ E(y) = E_0 \exp \left( \frac{y^2}{W_0^2} \right) \]

Where,

- \( 2W_0 = \text{Spot Size = Full Width of the Far-Field distribution} \)
- \( y \rightarrow \text{radius} \)
- \( E_0 \rightarrow \text{Field at Zero radius} \)
b) Propagation Modes in SM Fiber:

- Two independent, degenerate propagation modes
- Very similar, polarization planes are orthogonal
- Horizontal (H) and Vertical (V) polarization

* Birefringence *

a) Ideal fiber ⇒ Perfect rotational symmetry
   \( k_x = k_y \)

b) Practically \( k_x \neq k_y \)
   \( \rightarrow \) Lateral stress, non-circular cores

The mode propagate with different phase velocity and difference \( \beta \) of their effective refractive indices is called fiber birefringence \( (B_f) \)

\[
B_f = n_y - n_x
\]

\[
\beta = k_0 \left( n_y - n_x \right)
\]

\[
k_0 = \frac{2\pi}{\lambda}
\]
*Fiber Beat Length*: \( L_p \)

\[ L_p = \frac{2\pi}{\beta} \]

\[ L_p = \frac{\lambda}{\beta} \]

**V-number [Normalized Frequency]**

This determines how many modes a fiber can support.

\[ V = \frac{2\pi a}{\lambda} \left( \frac{n_1^2 - n_2^2}{2} \right)^{1/2} \] \text{cut-off pt when } V \leq 2.405

\[ V = \frac{2\pi a}{\lambda} \cdot NA \]

**Total no of modes Supported in a Fiber**

\[ M = \frac{1}{2} \left( \frac{2\pi a}{\lambda} \right)^2 \left( n_1^2 - n_2^2 \right) \]

\[ M = \frac{V^2}{2} \]

a) At cut-off pt => optical power of mode is residing at cladding.

b) Far from cut-off pt => low fractional of avg power in cladding.
\[
\frac{P_{\text{clad}}}{P} = \frac{4}{3JM}
\]

\(P\rightarrow\) total optical power in a fiber

**Note:**

Power flow of cladding decreases as \(V\)-number increases.

\[
V_p \ (\text{phase velocity}) = \frac{\omega}{k}
\]

\[
V_g \ (\text{group velocity}) = \frac{\partial \omega}{\partial k}
\]

*Modes in planar guide:*

---

\[AB = S_1\]
\[CD = S_2\]

Phase front of downward travelling wave

Phase front of upward travelling wave
According to ray theory, allow any rays at angle $\varphi$ greater than $\varphi_c$.

But interference effect by certain discrete angles greater than or equal to $\varphi_c$.

In diagram, the ray incident at material at angle $\Theta$:

$$\Theta = \frac{\pi}{2} - \varphi_c$$

$\Rightarrow$ wave travels through the material will undergo ($\Delta$) phase shift:

$$\Delta = k_s \cdot s = n_1 \cdot k_s = n_1 \cdot \frac{2\pi s}{\lambda}$$

$s$ = distance the wave has traveled in material.

1) From pt A to pt B, Ray 1,

$$S_1 = \frac{d}{\sin \Theta}, \ 2 \text{ phase change (8)}$$

To determine its phase change:

$$AD = \left( \frac{d}{\tan \Theta} \right) - d \tan \Theta \quad [\text{distance from A to D}].$$

$$S_2 = \frac{AD}{\cos \Theta} = \left( \cos^2 \Theta - \sin^2 \Theta \right) \frac{d}{\sin \Theta} \quad [\text{distance from C to D}].$$

The requirement for wave propagation:

$$\frac{2\pi n_1}{\lambda} (S_1 - S_2) + 2\delta = 2\pi m.$$
Sub $S_1 + S_2$ in Equ. 0

$$\frac{2\pi n}{\lambda} \left\{ \frac{d}{\sin \theta} - \left[ \frac{(\cos^2 \theta - \sin^2 \theta) d}{\sin \theta} \right]^2 \right\} + 2\delta = 2\pi m$$

$$\sin \phi = \frac{2\pi n d \sin \theta}{\lambda} + \delta = \pi m$$

Phase shift ($\delta$) is given by

$$\delta = -2 \arctan \left[ \frac{\sqrt{\cos^2 \theta - \left( \frac{n_2^2}{n_1^2} \right)}}{\sin \theta} \right]$$

$$\frac{2\pi n d \sin \theta}{\lambda} = \pi m = 2 \arctan \left[ \frac{\sqrt{\cos^2 \theta - \left( \frac{n_2^2}{n_1^2} \right)}}{\sin \theta} \right]$$

$$\tan \left( \frac{\pi n d \sin \theta}{\lambda} - \frac{\pi m}{2} \right) = \left[ \frac{\sqrt{n_2^2 \cos^2 \theta - n_2^2}}{n_1 \sin \theta} \right]$$

Note:

Only waves that have those angles $\theta$ which satisfy the condition will propagate in the $d$ wave guide.